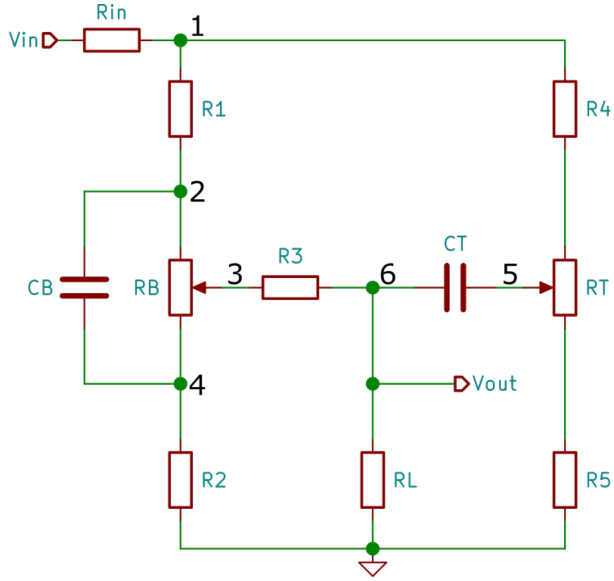
Circuit Analysis of Baxandall Tone Control

Variation: Passive, single bass capacitor, single treble capacitor

To find the frequency response of the circuit, the ratio $\frac{V_{out}}{V_{in}}$ needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



Nodal Analysis

Node #1: Treble potentiometer R_T is modeled as resistors R_{T1} and R_{T2}, connected at the wiper. The values of R_{T1} and R_{T2} are then increased to include series resistors R_4 and R_5 respectively. Using Kirchhoff's current law (KCL),

$$v_1(\tfrac{1}{R_{in}} + \tfrac{1}{R_{T1}} + \tfrac{1}{R_1}) + v_2(-\tfrac{1}{R_1}) + v_5(-\tfrac{1}{R_{T1}}) = \tfrac{V_{in}}{R_{in}}$$

Node #2: Bass potentiometer R_B is modeled as two resistors, R_{B1} and R_{B2}, connected at the wiper.

$$v_1(-\tfrac{1}{R_1}) + v_2(\tfrac{1}{R_1} + \tfrac{1}{R_{B1}} + j\omega C_B) + v_3(-\tfrac{1}{R_{B1}}) + v_4(-j\omega C_B) = 0$$

Node #3:

$$v_2(-\tfrac{1}{R_{B1}}) + v_3(\tfrac{1}{R_{B1}} + \tfrac{1}{R_{B2}} + \tfrac{1}{R_3}) + v_4(-\tfrac{1}{R_{B2}}) + V_{out}(-\tfrac{1}{R_3}) = 0$$

Node #4:

$$v_2(-j\omega C_B) + v_3(-\tfrac{1}{R_{B2}}) + v_4(\tfrac{1}{R_{B2}} + \tfrac{1}{R_2} + j\omega C_B) = 0$$

Node #5:

$$v_1(-\tfrac{1}{R_{T1}}) + v_5(\tfrac{1}{R_{T1}} + \tfrac{1}{R_{T2}} + j\omega C_T) + V_{out}(-j\omega C_T) = 0$$

Node #6:

$$v_3(-\tfrac{1}{R_3}) + v_5(-j\omega C_T) + V_{out}(\tfrac{1}{R_3} + \tfrac{1}{R_L} + j\omega C_T) = 0$$

Matrix Form

There are 6 node equations with 6 node voltage variables. These can be stated in matrix form

$$Ax = b$$

where A is a 6 x 6 matrix of the coefficients (admittances), x is a column vector of the variables (node voltages), and b is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix} \frac{1}{R_{in}} + \frac{1}{R_{T1}} + \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & -\frac{1}{R_{T1}} & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 & -\frac{1}{R_3} \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 & 0 \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T & -j\omega C_T \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T & \frac{1}{R_3} + \frac{1}{R_L} + j\omega C_T \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ V_{out} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{R_{in}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage, V_{out}, can now be found using Cramer's rule.

$$V_{out} = \frac{|A_6|}{|A|}$$

where matrix A_6 is formed by replacing the 6th column of A with the contents of b. Since b contains only one non-zero element, the determinant of A_6 is equal to that element multiplied by its cofactor.

$$\begin{aligned} |A_6| &= \frac{V_{in}}{R_{in}} C_{1,6} = \frac{V_{in}}{R_{in}} (-1)^{1+6} (M_{1,6}) \\ |A_6| &= -\frac{V_{in}}{R_{in}} M_{1,6} \end{aligned}$$

where $M_{1,6}$ is the determinant of A_6 with row 1 and column 6 removed.

$$M_{1,6} = \begin{vmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T \end{vmatrix}$$

Substituting into the equation for V_{out},

$$V_{out} = -\tfrac{V_{in}}{R_{in}} \frac{M_{1,6}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\tfrac{V_{out}}{V_{in}} = -\tfrac{1}{R_{in}} \frac{M_{1,6}}{|A|}.$$