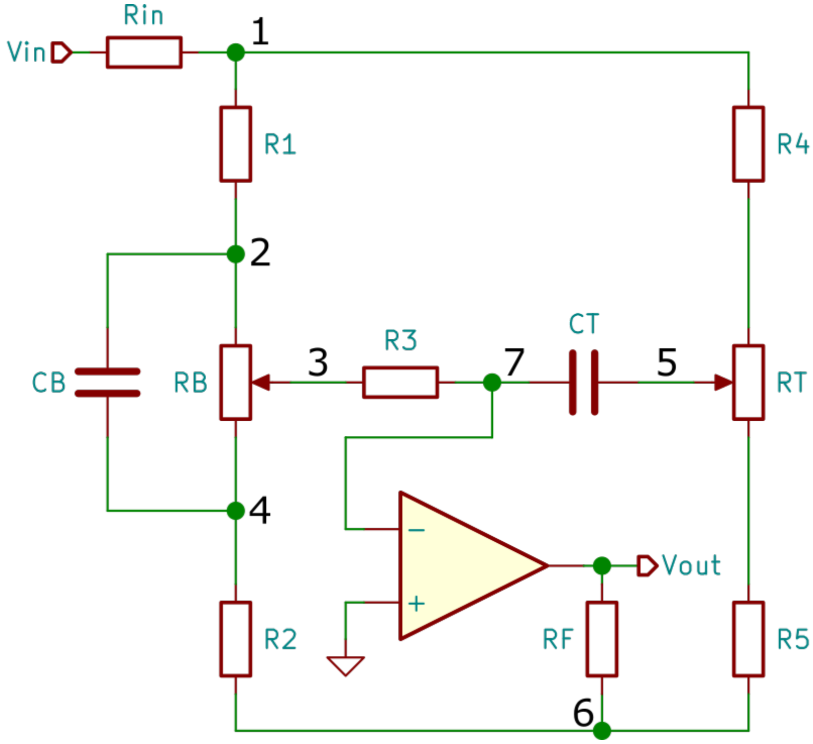
Analysis of James & Baxandall Tone Control Circuits

Variation: Active, single bass capacitor, single treble capacitor

To find the frequency response of the circuit, the ratio $\frac{V_{out}}{V_{in}}$ needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



Nodal Analysis

Node #1: Treble potentiometer R_T is modeled as resistors R_{T1} and R_{T2}, connected at the wiper. The values of R_{T1} and R_{T2} are then increased to include series resistors R_4 and R_5 respectively. Using Kirchhoff's current law (KCL),

$$v_1(\frac{1}{R_{in}} + \frac{1}{R_{T1}} + \frac{1}{R_1}) + v_2(-\frac{1}{R_1}) + v_5(-\frac{1}{R_{T1}}) = \frac{V_{in}}{R_{in}}$$

Node #2: Bass potentiometer R_B is modeled as two resistors, R_{B1} and R_{B2}, connected at the wiper.

$$v_1(-\frac{1}{R_1}) + v_2(\frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B) + v_3(-\frac{1}{R_{B1}}) + v_4(-j\omega C_B) = 0$$

Node #3: The positive and negative inputs of an ideal opamp are modeled as having no voltage difference between them. Since the positive input is grounded, the negative input (and therefore the connection to R_3 at node 7) is considered grounded as well.

$$v_2(-\frac{1}{R_{B1}}) + v_3(\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3}) + v_4(-\frac{1}{R_{B2}}) = 0$$

Node #4:

$$v_2(-j\omega C_B) + v_3(-\tfrac{1}{R_{B2}}) + v_4(\tfrac{1}{R_{B2}} + \tfrac{1}{R_2} + j\omega C_B) + v_6(-\tfrac{1}{R_2}) = 0$$

Node #5: As with R_3 above, the other side of C_T at node 7 is considered grounded since it is connected to the negative opamp input.

$$v_1(-\tfrac{1}{R_{T1}}) + v_5(\tfrac{1}{R_{T1}} + \tfrac{1}{R_{T2}} + j\omega C_T) + v_6(-\tfrac{1}{R_{T2}}) = 0$$

Node #6:

$$v_4(-\tfrac{1}{R_2}) + v_5(-\tfrac{1}{R_{T2}}) + v_6(\tfrac{1}{R_2} + \tfrac{1}{R_{T2}} + \tfrac{1}{R_F}) + V_{out}(-\tfrac{1}{R_F}) = 0$$

Node #7: The inputs of an ideal opamp are modeled as conducting no current.

$$v_3(-\tfrac{1}{R_3}) + v_5(-j\omega C_T) = 0$$

Matrix Form

There are 6 node equations with 7 node voltage variables. These can be stated in matrix form

$$Ax = b$$

where A is a 7 x 7 matrix of the coefficients (admittances), x is a column vector of the variables (node voltages), and b is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix} \frac{1}{R_{in}} + \frac{1}{R_{T1}} + \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & -\frac{1}{R_{T1}} & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 & 0 & 0 \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T & -\frac{1}{R_{T2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_2} & -\frac{1}{R_{T2}} & \frac{1}{R_2} + \frac{1}{R_{T2}} + \frac{1}{R_F} & -\frac{1}{R_F} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ V_{out} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{R_{in}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage, V_{out}, can now be found using Cramer's rule.

$$V_{out} = \frac{|A_7|}{|A|}$$

where matrix A_7 is formed by replacing the 7th column of A with the contents of b. Since b contains only one non-zero element, the determinant of A_7 is equal to that element multiplied by its cofactor.

$$\begin{aligned} |A_7| &= \frac{V_{in}}{R_{in}} C_{1,7} = \frac{V_{in}}{R_{in}} (-1)^{1+7} (M_{1,7}) \\ |A_7| &= \frac{V_{in}}{R_{in}} M_{1,7} \end{aligned}$$

where $M_{1,7}$ is the determinant of A_7 with row 1 and column 7 removed.

$$M_{1,7} = \begin{vmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 & 0 \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 & -\frac{1}{R_2} \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T & -\frac{1}{R_{T2}} \\ 0 & 0 & 0 & -\frac{1}{R_2} & -\frac{1}{R_{T2}} & \frac{1}{R_2} + \frac{1}{R_{T2}} + \frac{1}{R_F} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T & 0 \end{vmatrix}$$

Substituting into the equation for V_{out},

$$V_{out} = \frac{V_{in}}{R_{in}} \frac{M_{1,7}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_{in}} \frac{M_{1,7}}{|A|}.$$