

---

RE-ENGINEERING THE CRYBABY-STYLE WAH PEDAL

---

September 3, 2014

J.L.

## 1 BACKGROUND AND MOTIVATION

After learning more and more about feedback, I feel that my analysis on the wah-wah pedal in the book 'Science of Electric Guitars and Guitar Electronics' is not enough to explain how and why the 'original' wah pedal works as it does. Although the analysis based on the Miller theorem is valid, it does not reveal the fact that better insight of the circuit is obtained using feedback analysis. It turns out that the feedback analysis provides good design tools as well.

Although the wah-circuit is quite simple, there is so many cool things happening there! Personally I would vote this circuit as the most ingenious guitar effect of all time (if we don't count the effects which combine electronics and mechanics as the spring-reverb does). This writing aims to give a reasonably more in-depth view on the circuit and refer to some important design pointers both for DC biasing and AC processing in the middle of the analysis. For more information on the basic theory of second order resonators and filter synthesis, it is good to read the 6th edition of Microelectronic Circuits by Sedra and Smith. Along with the section on feedback stability, the chapter on filters, and especially the section on filter synthesis helps a lot to understand what is happening in the wah-circuit and how it has been designed.

In this writing I will use the essential parts of the theory to redesign the wah pedal. The basic circuit which the wah pedal is built upon is the very well known textbook example of the current-shunt feedback amplifier. If this is not a familiar configuration, please read my writing 'Basic Forms of Feedback In Voltage and Current Amplifiers', where a general method from Millman's book on Microelectronics is used for analysing feedback circuits. Unfortunately the feedback chapter in the book of Sedra and Smith gives a false idea on the different feedback parameters, mainly related to the reverse transmission factor  $\beta$  and loop gain  $T$ . The feedback analysis method presented by Millman is more accurate, but also more complex.

But for starters, let's take a look at the wah-circuit to be analysed and redesigned. Figure 1 is a reorganised schematic of the circuit shown in the US patent #3,530,224 : 'Foot controlled continuously variable preference circuit for musical instruments'. The redrawn version tries to put the circuit in a standard form where the basic configurations for biasing and feedback are more easily distinguished. The component values shown in Figure 1 are

---

directly taken from the patent documentation.

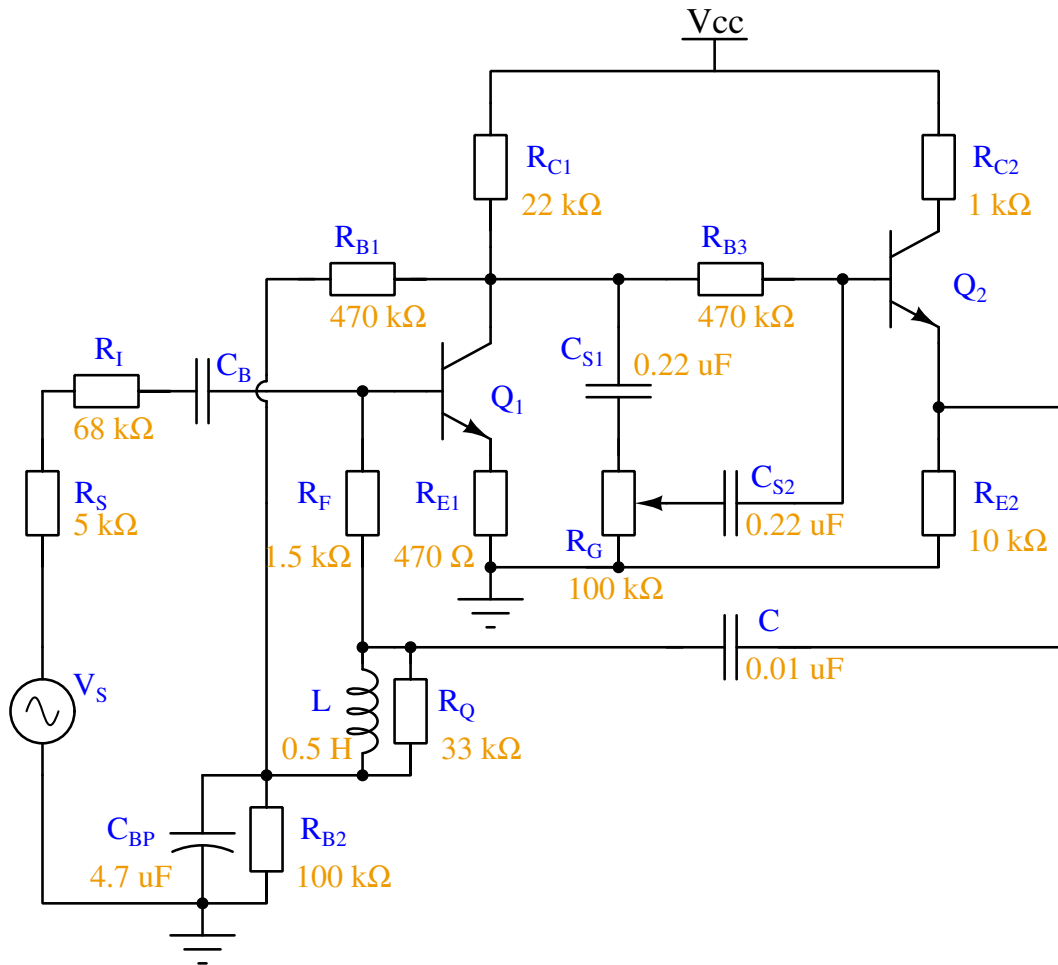


Figure 1: The patented wah-circuit design

As this writing will demonstrate later on, this circuit is based on the classic direct-coupled current-shunt feedback amplifier, the same configuration upon which the fuzz-face pedal is built as well. First we will analyse and give design tips for biasing the transistors, and then take on the challenge to find out the methods used for determining and designing the sweep-range of the variable filter.

## 2 DC ANALYSIS AND DESIGN

To ease out the DC analysis, it is a good habit to redraw the circuit by retaining only those components that affect the biasing of the transistors when operating voltage is applied to the circuit. The DC model of any circuit is obtained using the following simple rules: capacitors are open-circuited

and inductors are short-circuited. Applying this into the schematic of Figure 1, the circuit of Figure 2 is recovered. The configuration has been shuffled about a little to clearly show the basic biasing configurations involved around  $Q_1$  and  $Q_2$ .

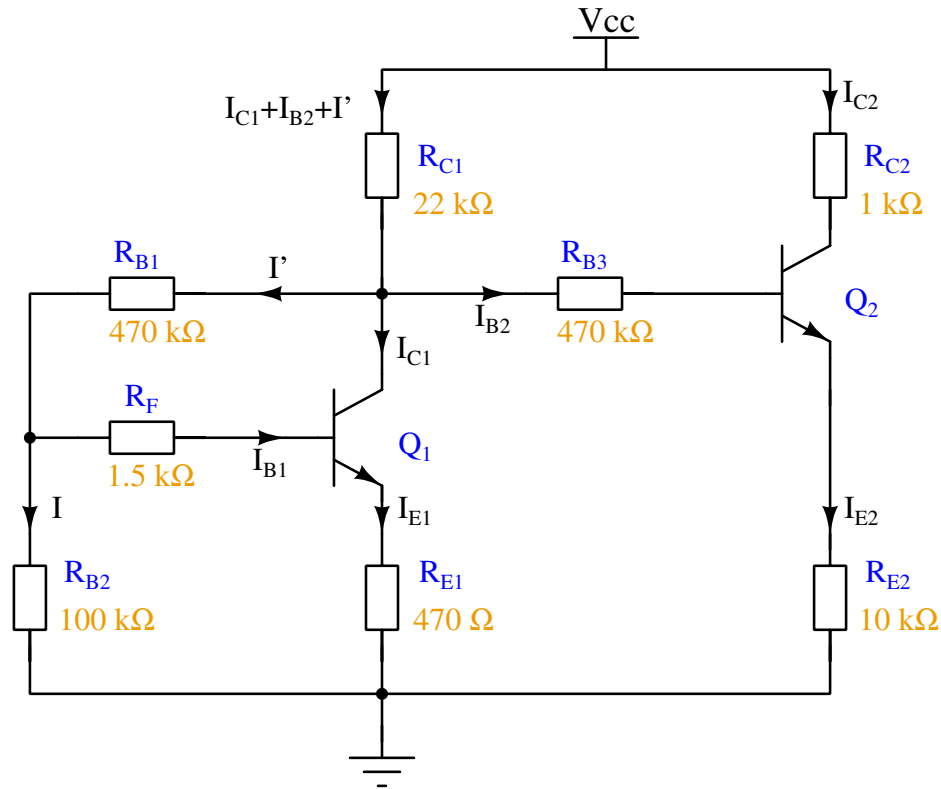


Figure 2: DC part of the wah-circuit

Since the transistors are connected to each other using direct coupling, the biasing of the first stage affects the second stage biasing and vice versa. However, if the current at the base of  $Q_2$  is designed small enough, the bias design for the two transistors can be done separately. This can be done for example by using a high current gain device as the transistor  $Q_2$ .

When examining the transistor stages separately, one can see that the biasing of the first stage is using the standard collector-to-base feedback bias technique, which is also used in the Big Muff  $\pi$  effect, for example. Only difference is the small 1.5 kilohm resistor in series with the transistor's base, but since it is 'so small', it can be assumed to be absorbed into the transistor's internal input resistance and therefore neglected from the biasing analysis. Hence, the DC bias currents of  $Q_1$  are fixed using resistors  $R_{B1}$  and  $R_{B2}$ .

## 2.1 Biasing of $Q_1$

For more indepth analysis, the bias configuration of the first gain stage is drawn independently in Figure 3, which includes the notation for all the necessary currents and voltage nodes.

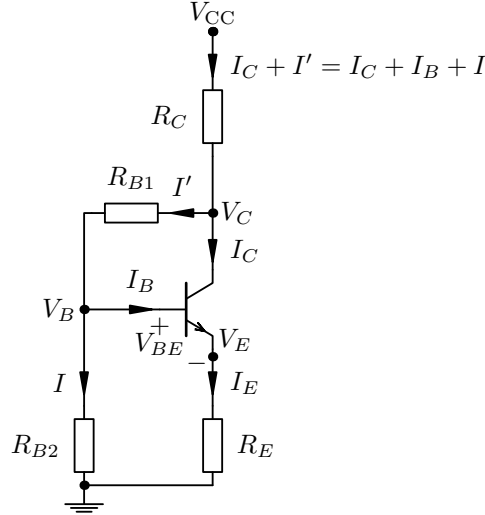


Figure 3: A collector-to-base bias arrangement with voltage divider at the base and emitter resistor

This configuration is completely covered with two branch current equations:

$$V_{CC} = (I_B + I_C)R_C + I(R_C + R_{B1}) + I_B R_{B1} + V_{BE} + (I_B + I_C)R_E$$

$$I R_{B2} = V_{BE} + (I_B + I_C)R_E.$$

Combining these two equations we get

$$V_{CC} = \left(1 + \frac{R_E}{R_{B2}}\right) (I_B + I_C)R_C + \left(1 + \frac{R_E}{R_{B2}}\right) I_B R_{B1} + I_C R_E \frac{R_{B1}}{R_{B2}} + (I_B + I_C)R_E + \frac{V_{BE}}{R_{B2}}(R_{B1} + R_{B2} + R_C)$$

and the collector current can be solved from here,

$$I_C = \beta_F \frac{V_{CC} - \frac{V_{BE}}{R_{B2}}(R_{B1} + R_{B2} + R_C)}{\left(1 + \frac{R_E}{R_{B2}}\right) [R_{B1} + R_C(\beta_F + 1)] + R_E \left(\beta_F \frac{R_{B1}}{R_{B2}} + \beta_F + 1\right)}.$$

The equation for collector current  $I_C$  can be easily converted into base current  $I_B$  and emitter current  $I_E$  using the well known transform factors consisting of  $\beta_F$ .

For further analysis one can solve equation for the collector voltage  $V_C$ . According to the Kirchhoff's current rule, the current equation for the voltage node  $V_B$  is:

$$I' = I + I_B = I + \frac{I_E}{\beta_F + 1}, \quad (1)$$

and for the voltage node  $V_C$ :

$$I'_C = I_C + I' = I_E \frac{\beta}{\beta_F + 1} + I'. \quad (2)$$

After the current equations are written down for each voltage node, the next step is to express the currents using the supply voltage  $V_{CC}$  and the node voltages. The currents appearing in the current equations can be expressed with respect to the node voltage as:

$$I'_C = \frac{V_{CC} - V_C}{R_C} \quad ; \quad I_E = \frac{V_B - V_{BE}}{R_E} \quad ; \quad I' = \frac{V_C - V_B}{R_{B1}} \quad ; \quad I = \frac{V_B}{R_{B2}}$$

and after substituting these voltage equations to the current equations,

$$\begin{aligned} \frac{V_{CC} - V_C}{R_C} &= \frac{V_B - V_{BE}}{R_E} \frac{\beta}{\beta + 1} + \frac{V_C - V_B}{R_{B1}} \\ \frac{V_C - V_B}{R_{B1}} &= \frac{V_B}{R_{B2}} + \frac{V_B - V_{BE}}{R_E} \frac{1}{\beta + 1}. \end{aligned}$$

These two equations can be rearranged into a matrix equation from where the node voltages can be solved systematically:

$$\begin{bmatrix} \frac{\beta}{\beta + 1} \frac{1}{R_C} + \frac{1}{R_{B1}} & \frac{\beta}{\beta + 1} \frac{1}{R_E} - \frac{1}{R_{B1}} \\ -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{\beta + 1} \frac{1}{R_E} \end{bmatrix} \times \begin{bmatrix} V_C \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{V_{CC}}{R_C} + \frac{\beta}{\beta + 1} \frac{V_{BE}}{R_E} \\ \frac{1}{\beta + 1} \frac{V_{BE}}{R_E} \end{bmatrix}.$$

From the matrix equation one can calculate the analytic expression to verify the collector voltage with the chosen bias values:

$$V_C = \frac{V_{CC}[R_E(\beta_F + 1)(R_{B1} + R_{B2}) + R_{B1}R_{B2}] + V_{BE}R_C[R_{B2}(\beta_F + 1) + \beta_F R_{B1}]}{\frac{\beta_F}{\beta_F + 1}[R_E(\beta_F + 1)(R_{B1} + R_{B2}) + R_{B1}R_{B2}] + R_C(\beta_F + 1)(R_E + R_{B2})}.$$

It is a bit lengthy equation, but handy when doing modifications to the component values.

Now we know how to analyse this bias arrangement, next challenge is to present guidelines for making modifications to the design. A typical rule of thumb is that the current flowing in the base bias divider should be 1/10 (one tenth) of the collector current  $I_C$ . If the typical current gain factor  $\beta_F$  of

the transistor is 100, this leaves another 1/10 headroom for base current  $I_B$  against any deviation in the  $\beta_F$  due to temperature changes or manufacturing inconsistencies. The headroom in the base current is needed, for example, if momentarily due to a change in temperature  $\beta_F$  drops, and base starts drawing more current than in a laboratory environment.

Hence, we try to follow the '1/10' rule. A reasonable, but approximate, choice for the design equation would be

$$\frac{I_C R_E V_{BE}}{R_{B2}} \approx \frac{I_C}{10},$$

where the factor of 10 comes from the '1/10' rule. From here it follows that

$$R_{B2} \approx 10 \left( R_E + \frac{V_{BE} R_C}{V_{CC} - V_C} \right), \quad (3)$$

if expressing it using the collector voltage  $V_C$ . Here we are assuming that the collector current can be determined by

$$I_C \approx \frac{V_{CC} - V_C}{R_C},$$

which is not exactly true, but close enough if the current in the base voltage divider is one tenth of the collector current.

After the value for  $R_{B2}$  has been determined, then

$$R_{B1} = \frac{R_C(\beta_F + 1)[V_C(R_E + R_{B2}) - V_{BE}R_{B2}] - \left( V_{CC} - \frac{\beta_F}{\beta_F + 1}V_C \right) R_E(\beta_F + 1)R_{B2}}{\left( V_{CC} - \frac{\beta_F}{\beta_F + 1}V_C \right) [R_E(\beta_F + 1) + R_{B2}] + V_{BE}R_C\beta_F}. \quad (4)$$

When considering AC gain, it would be best to bypass the emitter resistor with a capacitor, then it would also be possible to use the standard feedback analysis to this circuit. Without a bypass capacitor, there exists two feedback paths (voltage and current feedback) from output to input. This combination of two feedback paths complicates the AC analysis considerably.

## 2.2 Biasing of $Q_2$

The biasing configuration of the second transistor can be identified as the fixed bias circuit with added emitter resistor to increase the bias stability. This configuration is drawn separately into Figure 4. Typically  $V_x = V_{CC}$ , but in the direct coupling scheme  $V_x$  is the collector voltage of the first transistor

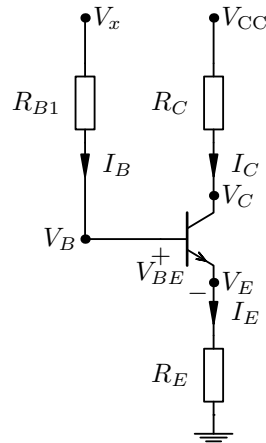


Figure 4: A fixed-bias configuration

stage. Using the notations introduced in Figure 4, the base current can be calculated from the equation

$$I_B = \frac{V_x - V_{BE}}{R_{B1} + R_E(B_F + 1)}.$$

When adapted to the wah-circuit,

$$I_{B2} = \frac{V_{C1} - V_{BE2}}{R_{B3} + R_{E2}(B_{F2} + 1)}.$$

When the base current is known, the collector and emitter currents can be found using the familiar current relations for transistor pins. The emitter voltage of the second stage would be

$$V_{E2} = R_{E2}(B_{F2} + 1)I_{B2}.$$

If a high gain transistor is used, the base current  $I_{B2}$  is minimal compared to  $I_{C1}$  and the bias design of  $Q_1$  and  $Q_2$  can be done separately.

### 3 AC ANALYSIS AND DESIGN

To aid the identification of the key components in the AC analysis, the wah-circuit is simplified to the form shown in Figure 5. In this reduced schematic, the DC blocking capacitors have been assumed to behave as short circuits and the relatively large biasing resistors are taken as open circuits.

The general configuration is easily identified as a direct-coupled transistor pair with a current-shunt feedback, which modifies the transistor amplifier into a current amplifier. The general current-shunt configuration applied to the wah-circuit is shown in Figure 6. The current-shunt configuration de-



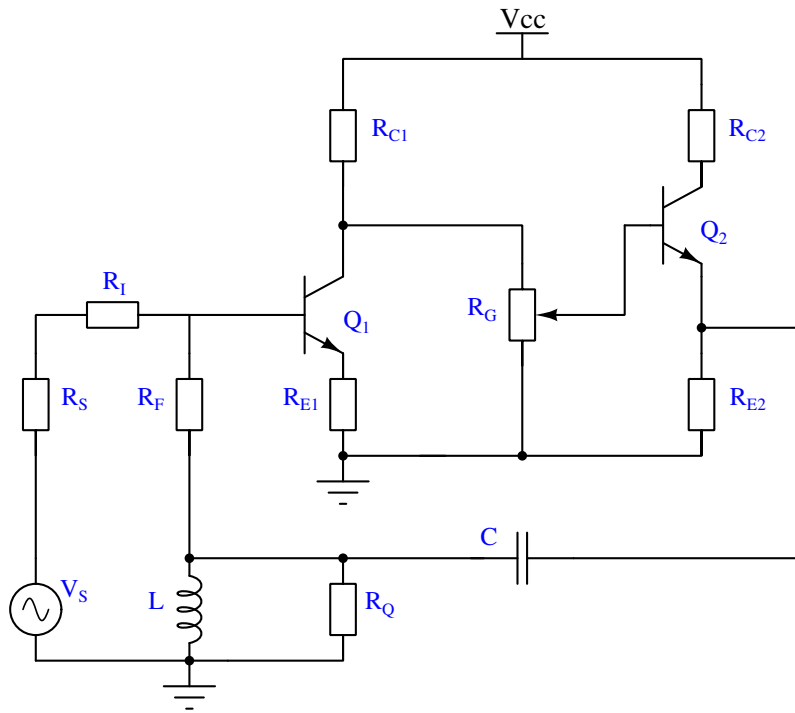


Figure 5: Simplified wah-circuit for AC signals

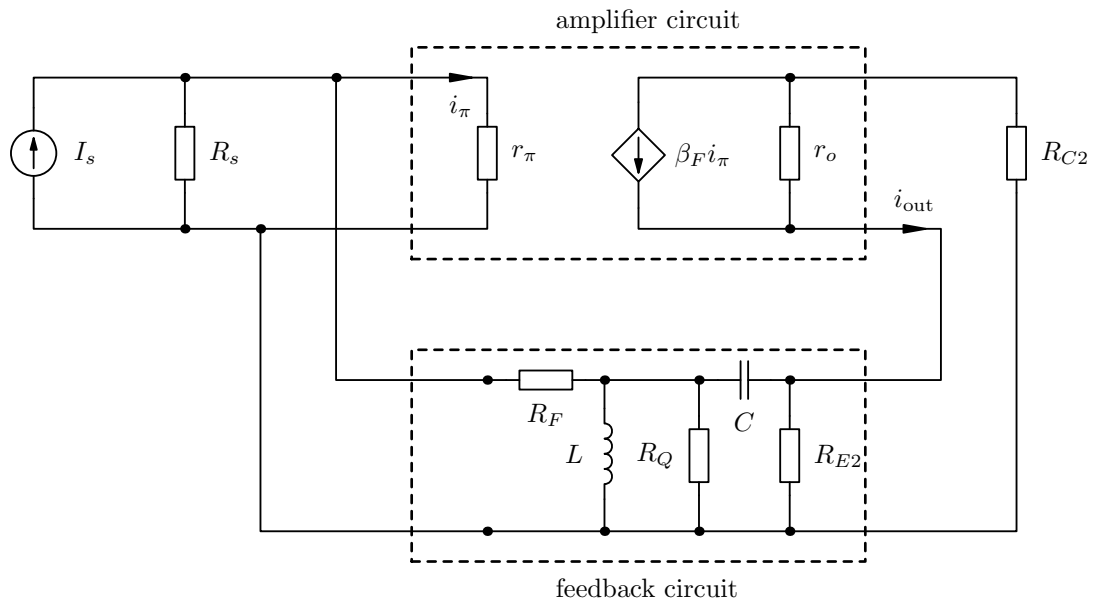


Figure 6: The current-shunt feedback configuration of the wah-circuit

creases the input impedance and increases the output impedance so that the input draws more current from the preceding device and introduces loading to some extent. The benefit of the current-shunt configuration in guitar effects is that the output looks more like a guitar output than a normal voltage amplifier.

There seems to be a frequency dependent thingy, consisting of  $L$ ,  $C$  and  $R_Q$ , in the feedback loop. This clearly has something to do with the frequency-sweeping properties of the wah-effect. So, let's identify that thingy first.

### 3.1 Parallel resonator and its applications as a filter

A short theoretical overview is in order before we dig into the actual wah-effect. The essential element which is part of the moving filter effect of the wah-circuit is the parallel resonator, which is drawn in its basic form in Figure 7. The resonator is a circuit of the second order, which means that the transfer function describing the frequency dependency has a second order equation in the denominator. In another words the circuit has two poles, which determine the resonance properties of the circuit.

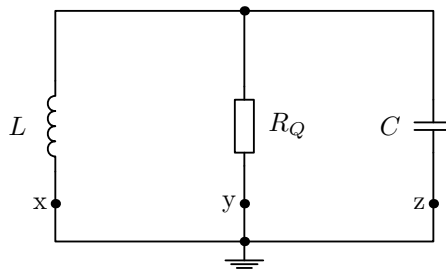


Figure 7: A parallel RLC resonance circuit

According to the basic theory of RLC circuits, the parallel RLC circuit has a resonance peak at

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}},$$

where  $\omega_0$  refers to angular frequency and  $f_0$  is the 'human readable' frequency. and the quality factor  $Q$  describing the sharpness of the peak is defined as

$$Q = \sqrt{\frac{C}{L}} R_Q.$$

The shape of the transfer function at left side of the resonance peak is controlled by the inductor, which impedance grows with frequency. Likewise, the right side of the transfer curve is controlled by the capacitor, which impedance decreases with frequency. The parallel resonator configuration can be used in filter applications as well by feeding a signal into the resonator through one of the three branches.

Surprisingly there exists a common theory, which states that any of the three grounded branches of the circuit can be used for injecting the input signal

into the circuit. The 'injection' is done by disconnecting the branch from the ground and applying the signal voltage into the disconnected end where the nodes labelled as  $x$ ,  $y$  or  $z$  in Figure 7 denote the location of injection.

Figure 8 depicts the idea of signal injection. In this type of transformation

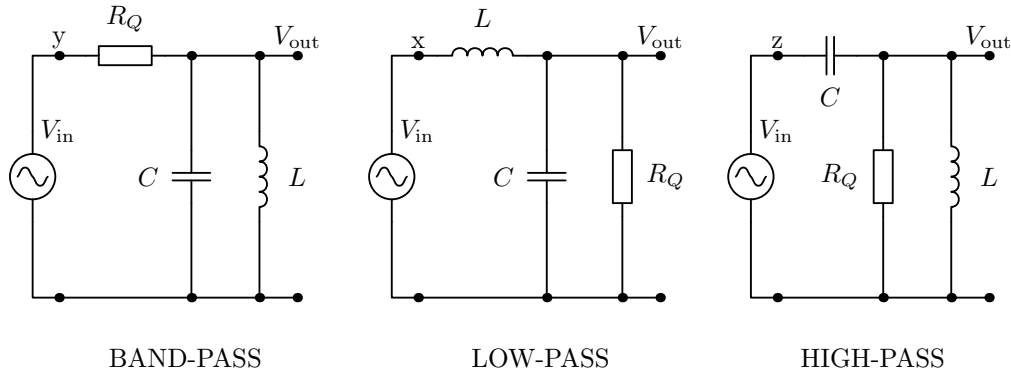


Figure 8: Parallel resonance circuit used for filter synthesis

the resonance frequency  $\omega_0$  and the quality factor  $Q$  of the resonator will not change. However, depending on which branch is disconnected, the resonator realises a different filter function. When the signal is fed through the inductor, the resonator behaves as a low-pass filter. When the signal is fed through the capacitor, the resonator behaves as a high-pass filter.

Now the resonator configuration of the wah-circuit has been identified as a second order high-pass filter, which is placed in the feedback loop. If the quality factor of the filter is larger than 1, the frequency response of the filter will show a peak at the vicinity of the resonant frequency.

The transfer function of the resonator high-pass circuit has the general form

$$H(s) = \frac{As^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}, \quad (5)$$

where the constant  $A$  acts as a gain factor, which can be used to adjust the level of the high-pass transfer curve at the pass-band. Figure 9 depicts the typical form of the high-pass function and effect of the gain factor  $A$ . These curves need to be demonstrated, because they will act as a reference when evaluating some of the transfer curves obtained from the wah-circuit.

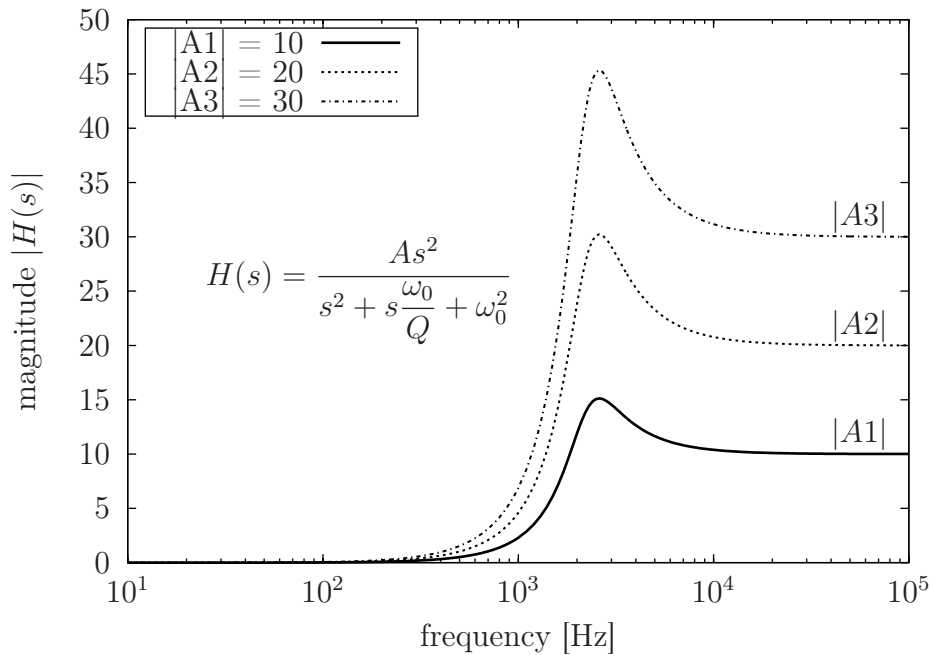


Figure 9: Second-order high-pass function with selected gain factors

### 3.2 Evaluating the general feedback factors of the wah-circuit

Let's move slowly towards the actual topic, the AC analysis of the wah-circuit. Even after the simplifications made in Figure 5, the small-signal model of the wah-circuit has eighth distinct voltage nodes. Clearly this circuit has too many components to start calculating exact equations for any of the feedback parameters. However, numerical analysis is still possible, since the circuit can be described by a matrix equation based on the voltage nodes. Because there are eighth voltage nodes in the circuit model, the matrix equation will contain a 8x8 admittance matrix. From the small-signal model one can construct a matrix equation, which can be analysed numerically in Octave or Matlab.

Basically we would need to construct two different small-signal models to evaluate all the feedback factors  $t_{11}$ ,  $t_{12}$ ,  $t_{21}$ ,  $t_{22}$  and the reverse transmission factor  $\beta$  and the return ratio  $T$ , which is also known as the loop gain. Since I have done the analysis for all of these parameters, I know that in this case the most meaningful design parameter for the wah-circuit is the loop gain  $T$ . Therefore, in this context we will only show how to evaluate the loop gain.

The small-signal model for evaluating the loop gain is drawn in Figure 10 and it is based on the AC configuration shown in Figure 5. When evaluating the loop gain, the input source is set to zero, which in this case means

that the current source is replaced with an open circuit. In the small-signal

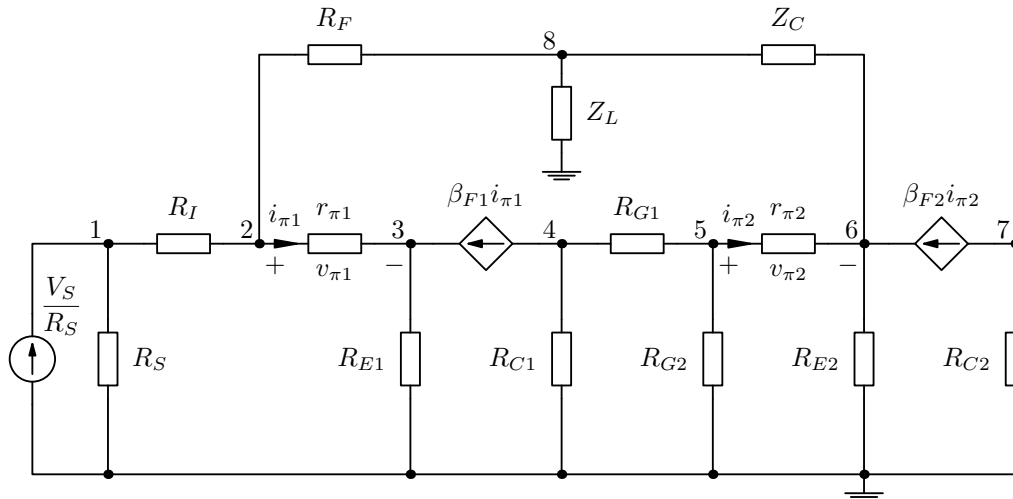


Figure 10: A simplified small-signal model of the wah-circuit

model the parallel connected inductor  $L$  and resistor  $R_Q$  have been referenced with a single impedance  $Z_L$ . Likewise, the impedance of capacitor  $C$  is referenced using the impedance  $Z_C$ . The potentiometer  $R_G$  controlling the current transfer to the second transistor is divided into two separate resistors  $R_{G1}$  and  $R_{G2}$  so that the current divider functionality of the potentiometer can be simulated.

When evaluating the loop gain, the feedback loop must be broken at some part of the circuit. The simplest place to brake the loop is always at the input of a transistor, because then there is no need to add any matched terminating impedance to the break point. The controlled current source acts as a natural loop breaker in this case. If there is more than one transistor in the circuit, normally it does not matter which transistor is chosen for the break point.

Normally ... yes, but unfortunately now in the wah-circuit it does matter. If the loop is broken at the input of  $Q_1$ , the gain provided by  $Q_1$  is lost from the loop and it will not create the Miller effect on the capacitor in the feedback loop. Breaking the loop at the input of  $Q_2$  at least partially retains the gain but the results seem different at first glance. Anyway, I just thought this was important to point out here.

To proceed with the analysis, we will choose the input of  $Q_1$  as the break point of the feedback loop and the the break point is identified by using the notation  $\beta_{F1}\hat{I}_{\pi1}$  for the 'broken' controlled source. This break point is chosen, because the resulting graph is more suitable for additional analysis.

We will use the systematic approach of analysing the circuit and set up a matrix equation, from where any of the node voltages shown in Figure 10 can be solved using Cramer's rule. The matrix equation derived directly from the small-signal model is written below.

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y_{21} & Y_{22} & Y_{23} & 0 & 0 & 0 & 0 & Y_{28} \\ 0 & Y_{32} & Y_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_{44} & Y_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_{54} & Y_{55} & Y_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_{65} & Y_{66} & 0 & Y_{68} \\ 0 & 0 & 0 & 0 & 0 & 0 & Y_{77} & 0 \\ 0 & Y_{82} & 0 & 0 & 0 & Y_{86} & 0 & Y_{88} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \beta_{F1} \hat{I}_{\pi 1} \\ -\beta_{F1} \hat{I}_{\pi 1} \\ 0 \\ \beta_{F2} i_{\pi 2} \\ -\beta_{F2} i_{\pi 2} \\ 0 \end{bmatrix}.$$

This matrix equation needs to be modified such that only one term related to the chosen controlled source is left to the current vector. We have chosen  $Q_1$  as the independent source, so that the term  $\hat{I}_{\pi 1}$  needs to be in the final solution. We choose to leave the current term on row 4 into the right side and move the rest of the current terms into the admittance matrix. The resulting matrix equation will then have two new admittance terms  $Y_{75}$  and  $Y_{76}$ , and the rest of the current terms are summed along with the existing admittance elements.

After these changes are made, the matrix equation looks as shown below:

$$\begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y_{21} & Y_{22} & Y_{23} & 0 & 0 & 0 & 0 & Y_{28} \\ 0 & Y_{32} & Y_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_{44} & Y_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_{54} & Y_{55} & Y_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_{65} & Y_{66} & 0 & Y_{68} \\ 0 & 0 & 0 & 0 & Y_{75} & Y_{76} & Y_{77} & 0 \\ 0 & Y_{82} & 0 & 0 & 0 & Y_{86} & 0 & Y_{88} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\beta_{F1} \hat{I}_{\pi 1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The admittance elements  $Y_{ij}$  referenced in the admittance matrix are written down as a separate listing to save some space in the actual matrix. The elements are:

$$\begin{aligned}
Y_{11} &= \frac{1}{R_s} + \frac{1}{R_I} & Y_{12} &= Y_{21} = -\frac{1}{R_I} \\
Y_{22} &= \frac{1}{R_I} + \frac{1}{R_F} + \frac{1}{r_{\pi 1}} & Y_{23} &= -\frac{1}{r_{\pi 1}} \\
Y_{28} &= Y_{82} = -\frac{1}{R_F} & Y_{32} &= -\frac{\beta_{F1} + 1}{r_{\pi 1}} \\
Y_{33} &= \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} & Y_{44} &= \frac{1}{R_{C1}} + \frac{1}{R_{G1}} \\
Y_{45} &= Y_{54} = -\frac{1}{R_{G1}} & Y_{55} &= \frac{1}{R_{G1}} + \frac{1}{R_{G2}} + \frac{1}{r_{\pi 2}} \\
Y_{56} &= -\frac{1}{r_{\pi 2}} & Y_{65} &= -\frac{\beta_{F2} + 1}{r_{\pi 2}} \\
Y_{66} &= \frac{\beta_{F2} + 1}{r_{\pi 2}} + \frac{1}{R_{E2}} + \frac{1}{Z_C} & Y_{68} &= Y_{86} = -\frac{1}{Z_C} \\
Y_{75} &= \frac{\beta_{F2}}{r_{\pi 2}} & Y_{76} &= -\frac{\beta_{F2}}{r_{\pi 2}} \\
Y_{77} &= \frac{1}{R_{C2}} & Y_{88} &= \frac{1}{R_F} + \frac{1}{Z_L} + \frac{1}{Z_C},
\end{aligned}$$

where in case of sinusoidal excitation  $s = j\omega$  and

$$Z_L = \frac{j\omega L R_Q}{j\omega L + R_Q} \quad \text{and} \quad Z_C = \frac{1}{j\omega C} \quad (6)$$

are the references for the capacitive and inductive impedance. From the matrix equation the loop gain is solved to be a function of the node voltages, namely

$$T(j\omega) = -\frac{V_2 - V_3}{r_{\pi 1}},$$

where the node voltages  $V_2$  and  $V_3$  are defined by the determinants as

$$V_2 = \beta_{F1} \frac{\det(V_2)}{\det(T)} \quad \text{and} \quad V_3 = -\beta_{F1} \frac{\det(V_3)}{\det(T)}.$$

An example Octave file is provided as an attachment to show in practise how to solve the matrix equation(s) numerically and find all the feedback parameters.

Now that we have a numerical way to evaluate the loop gain, we can create a magnitude plot as a function of frequency. This magnitude plot is illustrated in Figure 11. Comparing with Figure 9 it is easy to see that the shapes of the magnitude curves are almost identical. Hence, now we have a graphical way of determining  $T_0$  from the right side of the plot, where the frequency grows towards infinity. This  $T_0$  will be referred later on as the steady-state value of the loop gain. The following section shows its importance.

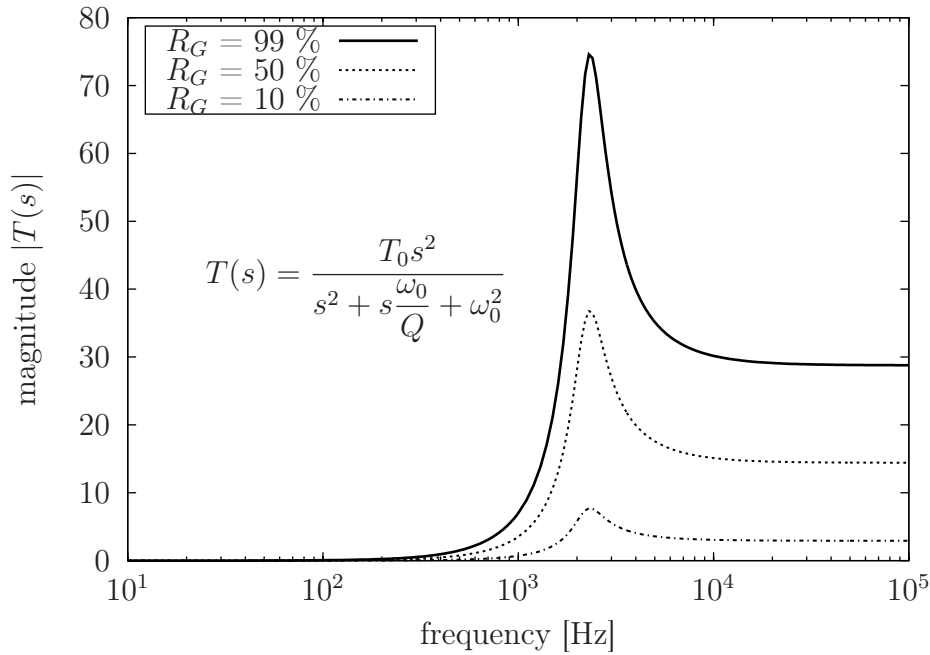


Figure 11: Loop gain  $T$  with selected divider values of  $R_G$

### 3.3 Loop gain $T$ is the key to success

So, since the graph of the loop gain was similar to the graph of the parallel resonator high-pass filter, we know that the transfer function of the loop gain can be written in the form of the parallel resonator high-pass filter

$$T(s) = \frac{T_0 s^2}{s^2 + s \frac{1}{CR_Q} + \frac{1}{LC}}, \quad (7)$$

where the quality factor  $Q$  and the squared angular frequency of resonance  $\omega_0^2$  have been explicitly written open. The value of the steady-state amplitude is set by  $T_0$ , which is found graphically or numerically when  $\omega \rightarrow \infty$ .

According to the standard theory of feedback circuits, the poles of any feedback amplifier are found by solving the equation

$$1 + T(s) = 0, \quad (8)$$

which is obtained by equating the denominator of the general feedback gain equation to zero. In this case, when the analysis only concerns sinusoidal signals (all practical signal can be presented as a sum of sine waves), the Laplace variable  $s$  can be replaced by the complex frequency  $j\omega$  and the loop



gain transfer function can be written as

$$T(s) = \frac{T_0 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = \frac{-T_0 \omega^2}{-\omega^2 + j\omega \frac{1}{CR} + \frac{1}{LC}}. \quad (9)$$

When inserting this equation into equation (8), we have

$$\frac{T_0 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = -1. \quad (10)$$

To find the values of  $s$  that make the equation have a zero value, the equation is set equal to zero,

$$s^2(1 + T_0) + s \frac{\omega_0}{Q} + \omega_0^2 = 0, \quad (11)$$

and the roots of the equation can be solved from there. As usual, we are interested only in sinusoidal signals, so then one can make the assignment  $s = j\omega$ . After solving the second-order equation using the quadratic formula, the roots that satisfy equation (11) are

$$j\omega = -\frac{\omega_0}{2Q(1 + T_0)} \pm j\sqrt{\frac{\omega_0^2}{1 + T_0} - \frac{\omega_0^2}{4Q^2(1 + T_0)^2}}. \quad (12)$$

This does not say much because of the imaginary term. However, the magnitude of this imaginary angular frequency is

$$|j\omega| = \omega = \sqrt{\frac{\omega_0^2}{4Q^2(1 + T_0)^2} + \frac{\omega_0^2}{1 + T_0} - \frac{\omega_0^2}{4Q^2(1 + T_0)^2}} = \frac{\omega_0}{\sqrt{1 + T_0}}, \quad (13)$$

which states that the resonance frequency is mapped from the original  $\omega_0$  by the factor of  $\frac{1}{\sqrt{1 + T_0}}$ . Also, from equation (10) one can take the term  $(1 + T_0)$  as a common multiplier and notice that the factors of  $s$  are in the 'standard' form:

$$(1 + T_0) \left( s^2 + s \frac{\omega_0}{Q(1 + T_0)} + \frac{\omega_0^2}{1 + T_0} \right) = 0. \quad (14)$$

Here we see directly that the quality factor  $Q$  is scaled by the factor  $\sqrt{1 + T_0}$ , i.e.,

$$Q = Q\sqrt{1 + T_0}. \quad (15)$$

So as a conclusion, the wah-circuit maps the nominal resonance frequency and the quality factor as

$$\omega_0 = \frac{\omega_0}{\sqrt{1 + T_0}} \quad ; \quad Q = Q\sqrt{1 + T_0}. \quad (16)$$

To visualise the result concerning the mapping of the resonance frequency Figure 12 is drawn using the component values of the wah circuit. With those values we plot a function

$$f(T) = \frac{\omega_0}{2\pi\sqrt{1+T}} = \frac{f_0}{\sqrt{1+T}} = \frac{2250}{\sqrt{1+T}}$$

with different values of  $T$  ranging from 0 to 50.

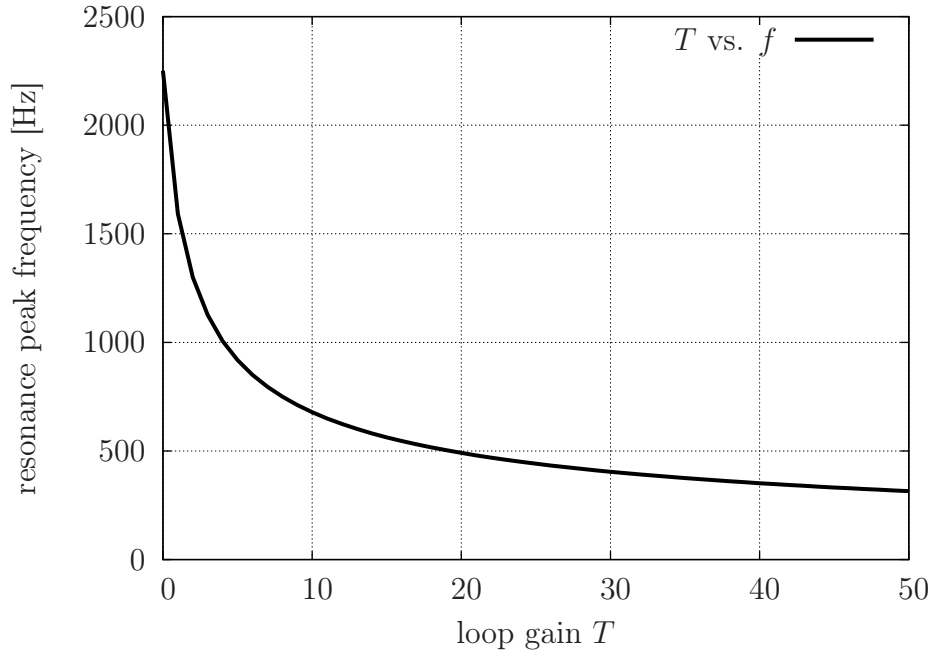


Figure 12: Frequency of the resonance peak as a function of the loop gain  $T$

Figure 12 indicates that the frequency of the resonance peak degrades rapidly when the loop gain  $T$  is close to zero. As  $T$  grows higher and higher, the change in the resonance peak frequency slows down. This gives a design pointer that the potentiometer controlling the loop gain should be designed to linearise the change in the resonance frequency. This will be addressed later on.

Still a few words about the loop gain maths. Taking the absolute value from both sides of equation (8) leads into a relation

$$|T(j\omega)| = 1, \quad (17)$$

which can be also written open to yield

$$\frac{T_0\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\omega \frac{\omega_0}{Q}\right)^2}} = 1. \quad (18)$$

When inserting the result  $\omega = \frac{\omega_0}{\sqrt{1+T_0}}$  it is noticed that when  $T_0 > 100$ , the equation (18) is satisfied nicely, but as the loop gain factor  $T_0$  decreases, the value of  $|T(s)|$  is clearly less than 1. The idea here is that the visual plots of the loop gain frequency response can reveal the resonance frequency directly when looking for the frequency where  $|T(s)|$  equals 1.

### 3.4 Exact formula for calculating $T_0$

So it seems that the value of  $T_0$  is the key for designing the sweep range of the wah effect. If one does not have the possibility to use mathematical software, it is kind of complicated to solve the steady-state loop gain  $T_0$ . Is there a simpler and faster way to obtain  $T_0$ ? Yes, but we need to derive the equation here first and then all you need to do is to plug in the component values to the resulting equation.

Since we know that  $T_0$  is obtained when  $\omega \rightarrow \infty$ , we can simplify the small-signal model to meet that specific condition. When the frequency approaches infinity, the capacitor becomes a short circuit and the inductor becomes an open circuit. Then we are left with the basic textbook example of a current-shunt feedback amplifier, where

$$R'_{E2} = \frac{R_{E2}R_Q}{R_{E2} + R_Q} \quad \text{and} \quad R'_{C1} = \frac{R_{C1}R_G}{R_{C1} + R_G}, \quad (19)$$

and in addition  $R'_S = R_S + R_I$ . These component merges leave us the small-signal model shown in Figure 13.

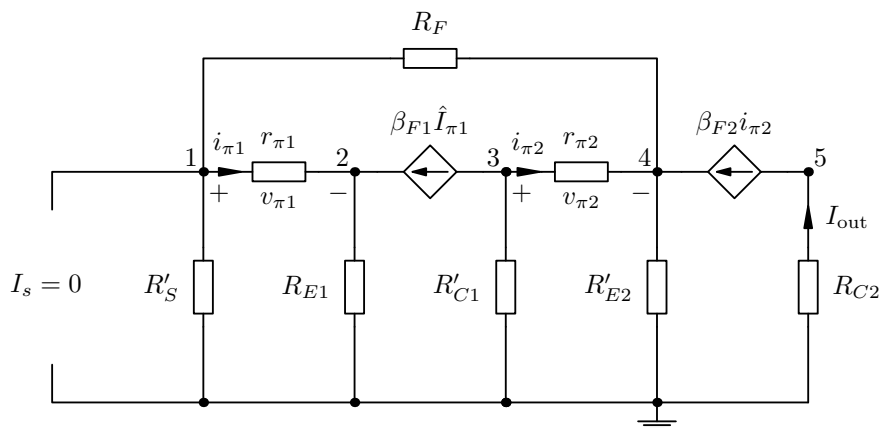


Figure 13: Even more reduced small-signal model for evaluating  $T_0$

The exact equation for  $T$  for the basic configuration of this current-shunt feedback amplifier is already evaluated in the text 'Basic Forms of Feedback

In Voltage and Current Amplifiers'. The only difference to that configuration is the added emitter resistor  $R_{E1}$ . Luckily, in this case, the effect of  $R_{E1}$  can be easily added to the equation derived from the basic model simply by replacing  $r_{\pi 1}$  with the series resistance  $r_{\pi 1} + R_{E1}(\beta_{F1} + 1)$ . After this replacement is done, the equation for the steady-state loop gain is

$$T_0 = \frac{R_P}{r_{\pi 1} + R_{E1}(\beta_{F1} + 1)} \frac{\beta_{F1} R'_{C1} R'_{E2} (\beta_{F2} + 1)}{(R'_{E2} + R_F + R_P)(R'_{C1} + r_{\pi 2}) + R'_{E2} (\beta_{F2} + 1)(R_F + R_P)},$$

where the parallel resistance term

$$R_P = \frac{R'_S [r_{\pi 1} + R_{E1}(\beta_{F1} + 1)]}{R'_S + r_{\pi 1} + R_{E1}(\beta_{F1} + 1)}$$

has been taken into use to simplify the equation.

Using the same component values when plotting Figure 11 the derived equation for  $T_0$  gives  $T_0 = 29.1$  with the maximum value of  $R_G$ . This clearly is the same result as indicated by the plot. Nice!

### 3.5 Evaluating the reverse transmission factor $\beta$

The analysis of the traditional current-shunt feedback amplifier proved that the reverse transmission factor  $\beta$  is determined by the passive network within the feedback loop. In the basic configuration the reverse transmission factor was defined by the current divider equation

$$\beta = \frac{R_{E2}}{R_{E2} + R_F}. \quad (20)$$

The feedback network in the wah-circuit is a bit more complex than two resistors, so the analysis needs a bit more work. One way to reduce the complex network into a current divider of two impedances is to apply the star-delta conversion to the feedback circuit. Figure 14 introduces the ideology of the star-delta conversion.

Without any excessive derivations, the formulae to convert the resistances (impedances) are

$$Z_P = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \quad ; \quad Z_Q = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \quad ; \quad Z_R = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}.$$

When applying these formulae into the feedback loop in the wah-circuit, we get a star configuration with following impedances:

$$Z_{E2} = \frac{Z_L R_{E2}}{Z_L + Z_C + R_{E2}} \quad ; \quad Z_F = \frac{Z_L Z_C}{Z_L + Z_C + R_{E2}} + R_F.$$

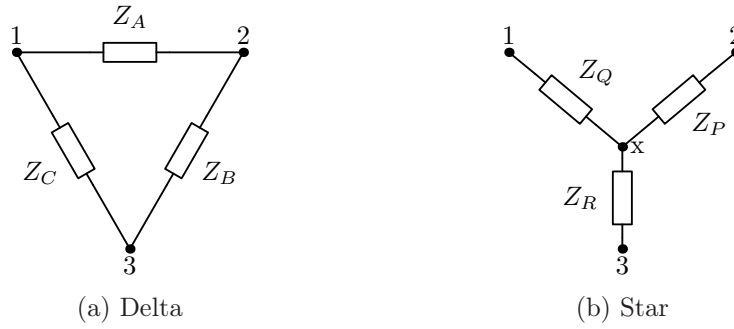


Figure 14: From a delta into a star conversion

These two impedances form the current divider in the wah-circuit. The current divider relation gives the  $\beta$  as

$$\beta = \frac{R_{E2}Z_L}{R_F(R_{E2} + Z_L + Z_C) + Z_L(R_{E2} + Z_C)}.$$

When the impedances  $Z_L$  and  $Z_C$  are written open, then

$$\beta = \frac{s^2 R_{E2} R_Q}{s^2 C L (R_F R_{E2} + R_F R_Q + R_{E2} R_Q) + s (C R_F R_{E2} R_Q + L R_F + L R_Q) + R_F R_Q}.$$

However, in this case the feedback factor does not tell much about the circuit itself. The current divider equation results in a high-pass form, but it is not reducible into the standard form of a second order filter. All the information we get from it is that the current divider also acts as some kind of a high-pass filter.

#### 4 DESIGNING THE POTENTIOMETER TO LINEARISE THE FREQUENCY SWEEP

As noted earlier, the resonance frequency changes with the loop gain following a square root law. As seen in Figure 12, the frequency sweep will pass over the high frequencies too fast if the loop gain  $T$  changes linearly. So we possibly want to linearise the sweep.

Firstly the answer is needed for the question: does the 'wah pot' change the loop gain  $T$  linearly? The answer is found by noting that the 'wah pot' forms a current divider with the input impedance of  $Q_2$ , which is  $r_{\pi 2} + R_{E2}(\beta_{F2} + 1)$ . If and when  $\beta_{F2}$  is greater than 100, the input impedance of  $Q_2$  is over 1 megohm. Then we can approximate the current going into the  $Q_2$  input as

$$I = \frac{R_G}{R_G + Z_I} \approx \frac{R_G}{Z_I}, \quad (21)$$

because  $Z_I > R_G$  so that  $R_G$  can be absorbed into  $Z_I$ . This 'proves' that the control relation between  $R_G$  and loop gain  $T$  is approximately linear with suitable values of total resistance  $R_G$ .

To linearise the frequency sweep,  $R_G$  should follow the curve of a square root function, which in certain limits is close to a logarithmic function. The resistance of  $R_G$  should change very slowly at lower resistances and more aggressively at the higher resistances. Figure 15 shows how a theoretical logarithmic control of the loop gain will result in a reasonably even frequency sweep.

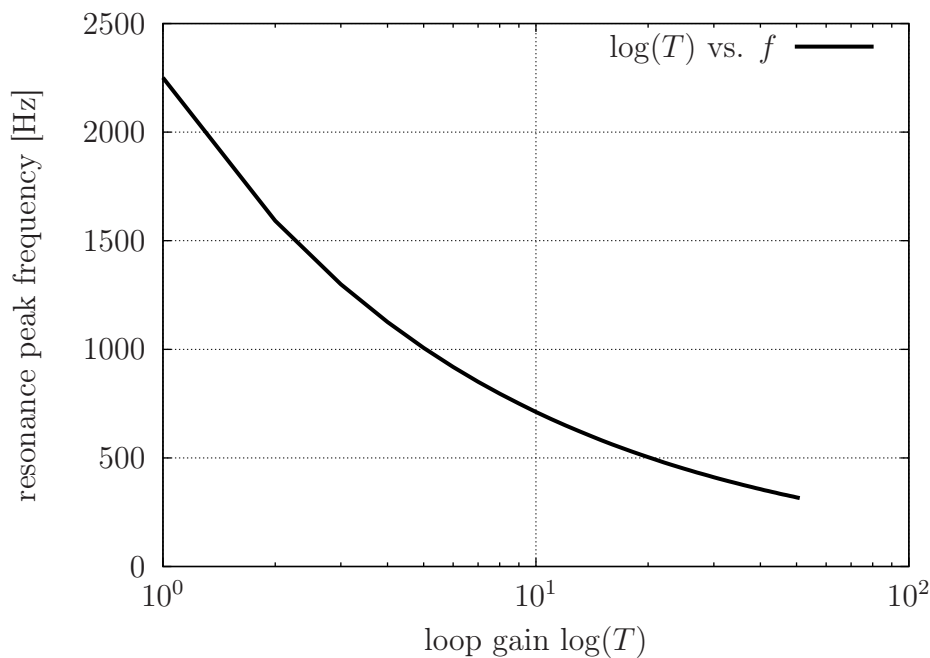


Figure 15: Linearising the frequency sweep with a logarithmic loop gain

Based on this discussion and the visualised results of logarithmic mapping of  $T$ , when selecting the potentiometer to a custom wah pedal build one should choose a logarithmic potentiometer with a total resistance clearly larger than  $R_{C1}$  (so it will not affect the gain of the first stage) and yet clearly smaller than the input impedance of the second transistor stage. Good values for the standard wah potentiometer are between 100 and 470 kilohms.

## 5 DESIGN PROCESS OF THE WAH EFFECT USING $T_0$

Just a short wrap-up on what we have learned in the previous analysis sections. The steady-state loop gain value  $T_0$  was found to have a direct relation

to the resonance frequency and also the sweep range of the wah-effect. Based on that, one can write down a few simple steps:

1. Choose the upper frequency limit  $f_0$  and lower limit  $f_l$  for the frequency sweep.
2. Select inductance  $L$  and capacitance  $C$  of the parallel resonator so that  $\omega_0 = 2\pi f_0$  becomes the resonance frequency
3. Determine the needed loop gain from the equation  $T_0 = \left(\frac{f_0}{f_l}\right)^2 - 1$
4. Tweak the component values  $\beta_{F1}, \beta_{F2}, R_{E1}, R_{C1}$  and possibly  $R_F$  in the equation of  $T_0$  to get close to the desired value
5. Select  $R_Q$  for the resonator to have a suitable quality factor, remember that  $T_0$  affects to the quality factor as  $Q = Q_0\sqrt{1 + T_0}$

This list is just to design the resonator and suitable sweep range. The total gain of at the output still needs to be tweaked to have a desired signal level. In addition to the components affecting the loop gain, the total gain can be enhanced by tweaking the input resistance  $R_I$ .

## 6 NON-IDEAL INDUCTOR IN THE PARALLEL RLC CIRCUIT

One common modification done to the wah-effect is to place a variable resistor to modify the quality factor of the circuit. This can be done two ways. Make  $R_Q$  variable, or add  $R_L$  in series with the inductor to increase its internal DC resistance as shown in Figure 16. The latter option is theoretically interesting and more complex so let's analyse that a little bit.

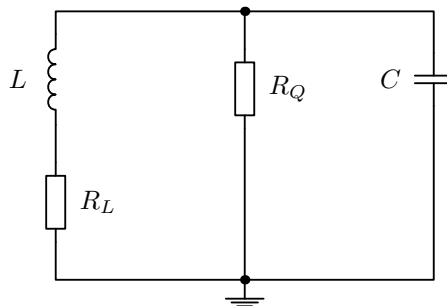


Figure 16: A parallel resonator with an non-ideal inductor

Typically most of the theoretical analysis seen in textbooks neglect the DC resistance of the inductor. Often this does not have any practical significance in the results, so it is safe to do so. When using inductors with huge amount of turns, the DC resistance grows to be significant and this is the case with the inductor in the wah-circuit. When placing an additional series resistor, then at latest we cannot just neglect that in the analysis.

In a parallel RLC circuit the current

$$I = VY = \frac{V}{Z},$$

as typically is according to Ohm's law. The impedance of an ideal parallel RLC circuit is

$$Z = \frac{V}{I} = \frac{1}{sC + \frac{1}{R_Q} + \frac{1}{sL}},$$

using the Laplace variable  $s$  to denote the complex frequency. When the inductor is not ideal, the impedance

$$Z = \frac{V}{I} = \frac{1}{sC + \frac{1}{R_Q} + \frac{1}{sL + R_L}} = \frac{1}{sC + \frac{1}{R_Q} + \frac{sL - R_L}{s^2L^2 + R_L^2}},$$

where the latter term is expanded using its complex conjugate to remove the 'complexity' from its denominator. When the simplification of the impedance equation is continued towards the standard form of transfer functions, we finally end up with an equation

$$Z = \frac{\frac{s}{C}(1 + Q_L^{-2})}{s^2 + \frac{s}{C}(1 + Q_L^{-2})\frac{R_L(1 + Q_L^2) + R_Q}{R_L(1 + Q_L^2)R_Q} + \frac{1}{LC} - \frac{R_L^2}{L^2}}, \quad (22)$$

where  $Q_L = \frac{\omega L}{R_L}$  is the quality factor of the inductor itself. It is important to note that  $Q_L$  depends on frequency, so it therefore has a different value for each frequency it is evaluated on. Additionally, we can assign a parallel resistance term

$$R' = \frac{R_L(1 + Q_L^2)R_Q}{R_L(1 + Q_L^2) + R_Q}, \quad (23)$$

to simplify the impedance equation into

$$Z = \frac{s \frac{(1 + Q_L^{-2})}{C}}{s^2 + s \frac{(1 + Q_L^{-2})}{CR'} + \frac{1}{LC} - \frac{R_L^2}{L^2}}. \quad (24)$$



From the simplified impedance equation, the resonance frequency and quality factor are now

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \quad \text{and} \quad Q = \frac{R' \sqrt{\frac{C}{L} - \left(\frac{CR_L}{L}\right)^2}}{1 + Q_L^{-2}}. \quad (25)$$

This result affects two things. Or maybe even three. Firstly, the resonance frequency term appears now to be lower by the term  $\frac{R_L^2}{L^2}$ . Secondly, but not as clearly seen, the quality factor degrades mostly because of the parallel connection between  $R_Q$  and the frequency mapped resistance  $R_L(1 + Q_L^2)$ .

The truth is that the resonance frequency is shifted, but not always lower. The resonance frequency is still evaluated as the frequency where the imaginary part of the denominator vanishes. In this case it is not the frequency determined by the term  $\frac{1}{LC} - \frac{R_L^2}{L^2}$  alone, but  $R_Q$  adds its flavours there as well. It is true that the resonance frequency is slightly shifted, but the direction has to be checked numerically as the frequency when the imaginary part of the denominator is zero.

The quality factor of the non-ideal circuit drops as the resistance  $R_L$  grows. This can be verified by checking what happens when  $R_L$  is large and small. When  $R_L$  is very small, then the term  $(1 + Q_L^2)$  will grow significantly (and  $1 + Q_L^{-2}$  will equal 1) and the effective value of  $R' = R_Q$ . When  $R_L$  grows larger, the parallel resistance of  $R_L$  and  $R_Q$  will be smaller than  $R_Q$  alone, so the quality factor decreases.

Not so simple to analyse, but the above gives the main points in a nutshell.

## 7 WHY THE OVERALL TRANSFER FUNCTION IS NOT HIGH-PASS BUT LOW-PASS?

When the gain of the wah-circuit is drawn as a function of frequency without the DC-blocking capacitors, one notices that the filter form is actually low-pass instead of high-pass. See for your self (Figure 17):

How is this possible? This is due to the fact that the high-pass filter has been placed within the feedback loop. Anything placed into the feedback loop is seen as inverted in the overall gain. The rationale here is that high frequencies will arrive back to the input with higher amplitude and will reduce the

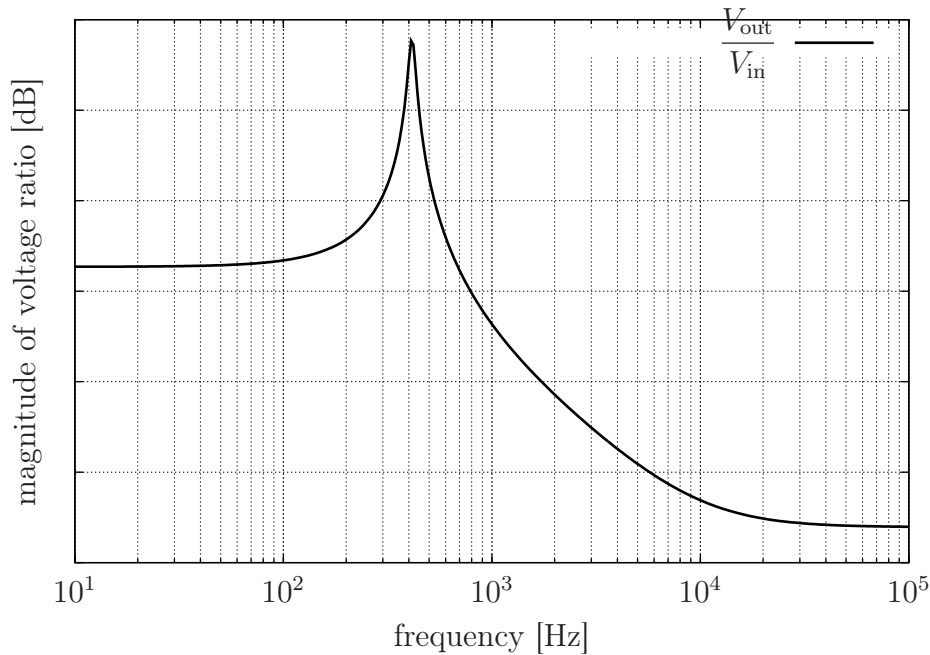


Figure 17: Overall gain in decibels as a function of frequency

original signal more effectively than the low frequencies. The frequency response of the complete circuit as shown in Figure 1 will be a combination of the high-pass filter formed by  $C_B$  and the low-pass filter formed by the feedback circuit. Hence, the output obtained from the wah resembles more like a band-pass filter.

## 8 CONCLUSIONS AND POSSIBLE MODS BASED ON THE ANALYSIS RESULTS

When building the wah-pedal as a DIY-project, a common problem has been to find a sufficiently large inductor. Based on the given analysis, there are at least two options to try to overcome this problem. The third modification suggestion is more theoretical and requires recalculation of the effect of  $T_0$  in the filter behaviour. The three modification suggestions are:

1. To be able to use a smaller inductor, scale the values of  $L$ ,  $C$  and  $R_Q$  so that  $\omega_0$  and  $Q$  remain the same. One example for this approach is to take  $L = 0.05$  H,  $C = 0.1\mu\text{F}$  and  $R_Q = 3300\Omega$ . However, then the overall gain will be reduced and this possibly needs to be compensated somehow. To increase the overall gain one can:

\* decrease or remove  $R_{E1}$

- \* increase  $R_{C1}$
  - \* increase  $R_F$
  - \* decrease  $R_I$
2. Again striving for a smaller inductor, do impedance scaling on the inductor alone and increase the loop gain considerably. This option will increase or decrease the sweep range, however.
  3. Try different filter types in the feedback loop. Especially the band-pass filter might be worth while trying out. However, the effect of loop gain to the band-pass configuration needs to be re-evaluated.

Please note that I have not tested these myself, these suggestions are based on the theoretical observations made in this text.

As a final conclusion, the following list explains the main purpose of each component in the wah-circuit.

- $R_I$  Connects in series with the output impedance of the connecting device, affects the input impedance and gain
  - $C_B$  DC blocking capacitor, but since having a small value, it is used also as a high-pass filter
  - $C_{BP}$  A bypass capacitor to create a ground point for AC signals, but preserve DC signal path
  - $R_{B1}$  Bias resistor for  $Q_1$
  - $R_{B2}$  Bias resistor for  $Q_1$
  - $R_{C1}$  Sets the collector current and with  $R_{E1}$  sets the gain of  $Q_1$  as  $Gain = \frac{R_{C1}}{R_{E1}}$
  - $R_{E1}$  Increases the input impedance of  $Q_1$  and sets the gain with  $R_{C1}$
  - $R_{B3}$  Bias resistor and current limiter for  $Q_2$
  - $R_G$  Controls the gain of the feedback loop
  - $C_{S1}$  Separates the signal path from the DC bias
  - $C_{S2}$  Separates the signal path from the DC bias
-

$R_{C2}$  Just a collector resistor in a common-emitter configuration. Not that meaningful in the big picture.

$R_{E2}$  Increases the input impedance of  $Q_2$ , part of the current divider in the feedback loop

$C$  With  $L$  sets the resonance frequency  $f_0$  and is part of the current divider in the feedback loop

$L$  With  $C$  sets the resonance frequency  $f_0$  and is part of the current divider in the feedback loop

$R_Q$  Controls the quality factor of the parallel resonator, affects the steady-state loop gain value  $T_0$

$R_F$  Is part of the current divider in the feedback loop, affects loop gain and total gain

## 9 APPENDIX A: EXAMPLE OCTAVE FILE FOR ANALYSIS

The code below should be saved as `wah.m` file and executed under Octave or Matlab using the command: `wah(90,10,100000,1)`

```
function z = wah(deviation, startfreq, stopfreq, mode)

% define the step size for frequency vector
% adder is the step for arithmetic series (mode = 1)
% multiplr is the step for geometric series (mode = 2)
adder = (10-1)/deviation;
multiplr = 10^(1/deviation);

% determine the exponent k according to input variable stopfreq
k = 0;
while 10^k < stopfreq
k = k + 1;
end;
% buffer to store the frequency and the magnitude data for plotting
PLOTBUF = [];

% buffers to hold magnitude (H) and frequency (X) data
HH=[];
KK=[];
JJ=[];
LL=[];
```

---

---

```

X=[];
%initial value for the start frequency
f = 1;

% loop over the given frequency range
for j = 0:k
while f <= (10^(j+1) - adder)
if (f >= (startfreq - 0.00001) && f <= stopfreq)
w = f*2*pi;

% Component values taken from the wah-circuit.
% The values for internal resistances rpi1 and rpi2 are guessed.
RI = 68000;
RS = 5000;
RF = 1500;
RQ = 33000;
L = 0.5;
C = 0.01e-6;
RE2 = 10000;
RE1 = 470;
RC1 = 22000;
RG1 = 50000;
RG2 = 50000;
RC2 = 1000;
rpi1 = 22000;
rpi2 = 22000;
BF1 = 200;
BF2 = 200;

% Values for evaluating T0 later on
RSc = RI + RS;
RE2c = RQ*RE2/(RQ+RE2);
RC1c = 100000*RC1/(100000 + RC1);

% Impedances of the reactive components
ZL = i*w*L*RQ/(i*w*L + RQ);
ZC = 1/(i*w*C);

% EVALUATION OF THE FEEDBACK PARAMETER t21
Y11 = 1/RS + 1/RI;
Y12 = Y21 = -1/RI;
Y22 = 1/RI + 1/RF + 1/rpi1;
Y23 = Y32 = -1/RF;
Y24 = -1/rpi1;
Y33 = 1/RF + 1/ZL + 1/(ZC + RE2);

```

---

```

Y42 = -(BF1+1)/rpi1;
Y44 = (BF1+1)/rpi1 + 1/RE1;
Y52 = BF1/rpi1;
Y54 = -BF1/rpi1;
Y55 = 1/RC1 + 1/RG1;
Y56 = Y65 = -1/RG1;
Y66 = 1/RG1 + 1/RG2 + 1/rpi2;

t21osV2 = [ Y21  Y23  Y24  0  0; ...
            0  Y33  0  0  0; ...
            0  0  Y44  0  0; ...
            0  0  Y54  Y55  Y56; ...
            0  0  0  Y65  Y66;];

t21osV4 = [ Y21  Y22  Y23  0  0; ...
            0  Y32  Y33  0  0; ...
            0  Y42  0  0  0; ...
            0  Y52  0  Y55  Y56; ...
            0  0  0  Y65  Y66;];

t21nim = [ Y11  Y12  0  0  0  0; ...
           Y21  Y22  Y23  Y24  0  0; ...
           0  Y32  Y33  0  0  0; ...
           0  Y42  0  Y44  0  0; ...
           0  Y52  0  Y54  Y55  Y56; ...
           0  0  0  0  Y65  Y66;];

I2 = -det(t21osV2)/(rpi1*(det(t21nim)));
I4 = -det(t21osV4)/(rpi1*(det(t21nim)));
t21 = I2 - I4;

% EVALUATION OF THE FEEDBACK PARAMETERS T and t12
Y11 = 1/RS + 1/RI;
Y12 = Y21 = -1/RI;
Y22 = 1/RI + 1/RF + 1/rpi1;
Y23 = -1/rpi1;
Y28 = Y82 = -1/RF;
Y32 = -(BF1+1)/rpi1;
Y33 = (BF1+1)/rpi1 + 1/RE1;
Y44 = 1/RC1 + 1/RG1;
Y45 = Y54 = -1/RG1;
Y55 = 1/RG1 + 1/RG2 + 1/rpi2;
Y56 = -1/rpi2;
Y65 = -(BF2+1)/rpi2;

```

---

$$Y66 = (BF2+1)/rpi2 + 1/RE2 + 1/ZC;$$

$$Y68 = Y86 = -1/ZC;$$

$$Y75 = BF2/rpi2;$$

$$Y76 = -BF2/rpi2;$$

$$Y77 = 1/RC2;$$

$$Y88 = 1/RF + 1/ZL + 1/ZC;$$

$$\begin{aligned} \text{TosV2} = & [ Y11 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & Y21 \quad Y23 \quad 0 \quad 0 \quad 0 \quad 0 \quad Y28; \dots \\ & 0 \quad Y33 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad Y54 \quad Y55 \quad Y56 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad Y65 \quad Y66 \quad 0 \quad Y68; \dots \\ & 0 \quad 0 \quad 0 \quad Y75 \quad Y76 \quad Y77 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad 0 \quad Y86 \quad 0 \quad Y88; ] \end{aligned}$$

$$\begin{aligned} \text{TosV3} = & [ Y11 \quad Y12 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & Y21 \quad Y22 \quad 0 \quad 0 \quad 0 \quad 0 \quad Y28; \dots \\ & 0 \quad Y32 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad Y54 \quad Y55 \quad Y56 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad Y65 \quad Y66 \quad 0 \quad Y68; \dots \\ & 0 \quad 0 \quad 0 \quad Y75 \quad Y76 \quad Y77 \quad 0; \dots \\ & 0 \quad Y82 \quad 0 \quad 0 \quad Y86 \quad 0 \quad Y88; ] \end{aligned}$$

$$\begin{aligned} \text{t12osV7} = & [ Y11 \quad Y12 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & Y21 \quad Y22 \quad Y23 \quad 0 \quad 0 \quad 0 \quad Y28; \dots \\ & 0 \quad Y32 \quad Y33 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad Y54 \quad Y55 \quad Y56 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad 0 \quad Y65 \quad Y66 \quad Y68; \dots \\ & 0 \quad 0 \quad 0 \quad 0 \quad Y75 \quad Y76 \quad 0; \dots \\ & 0 \quad Y82 \quad 0 \quad 0 \quad 0 \quad Y86 \quad Y88; ] \end{aligned}$$

$$\begin{aligned} \text{Tnim} = & [ Y11 \quad Y12 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & Y21 \quad Y22 \quad Y23 \quad 0 \quad 0 \quad 0 \quad 0 \quad Y28; \dots \\ & 0 \quad Y32 \quad Y33 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad Y44 \quad Y45 \quad 0 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad Y54 \quad Y55 \quad Y56 \quad 0 \quad 0; \dots \\ & 0 \quad 0 \quad 0 \quad 0 \quad Y65 \quad Y66 \quad 0 \quad Y68; \dots \\ & 0 \quad 0 \quad 0 \quad 0 \quad Y75 \quad Y76 \quad Y77 \quad 0; \dots \\ & 0 \quad Y82 \quad 0 \quad 0 \quad 0 \quad Y86 \quad 0 \quad Y88; ] \end{aligned}$$

$$I2 = BF1 * \det(\text{TosV2}) / (rpi1 * (\det(\text{Tnim})));$$

$$I3 = -BF1 * \det(\text{TosV3}) / (rpi1 * (\det(\text{Tnim})));$$

```

T = -(I2-I3);

% EVALUATION OF T0 with RG = 100000
RP = RSc*(rpi1 + RE1*(BF1+1))/( RSc + rpi1 + RE1*(BF1+1) );
T0 = (RP/(rpi1+RE1*(BF1+1)))*(BF1*(BF2+1)*RE2c*RC1c)/ ...
      ( (RF+RE2c+RP)*(rpi2+RC1c) + (BF2+1)*RE2c*(RF+RP) );

Io = -BF1*det(t12osV7)/(RC2*(det(Tnim)));
t12 = Io;

% EVALUATION OF THE FEEDBACK PARAMETER t11
Y11 = 1/RS + 1/RI;
Y12 = Y21 = -1/RI;
Y22 = 1/RI + 1/RF + 1/rpi1;
Y23 = -1/rpi1;
Y28 = Y82 = -1/RF;
Y32 = -1/rpi1;
Y33 = 1/rpi1 + 1/RE1;
Y44 = 1/RC1 + 1/RG1;
Y45 = Y54 = -1/RG1;
Y55 = 1/RG1 + 1/RG2 + 1/rpi2;
Y56 = -1/rpi2;
Y65 = -(BF2+1)/rpi2;
Y66 = (BF2+1)/rpi2 + 1/RE2 + 1/ZC;
Y68 = Y86 = -1/ZC;
Y75 = BF2/rpi2;
Y76 = -BF2/rpi2;
Y77 = 1/RC2;
Y88 = 1/RF + 1/ZL + 1/ZC;

t11osV7 = [ Y21  Y22  Y23  0  0  0  Y28;...
            0  Y32  Y33  0  0  0  0; ...
            0  0  0  Y44  Y45  0  0; ...
            0  0  0  Y54  Y55  Y56  0; ...
            0  0  0  0  Y65  Y66  Y68; ...
            0  0  0  0  Y75  Y76  0; ...
            0  Y82  0  0  0  Y86  Y88;];

t11nim = [ Y11  Y12  0  0  0  0  0  0; ...
           Y21  Y22  Y23  0  0  0  0  Y28;...
           0  Y32  Y33  0  0  0  0  0; ...
           0  0  0  Y44  Y45  0  0  0; ...
           0  0  0  Y54  Y55  Y56  0  0; ...
           0  0  0  0  Y65  Y66  0  Y68; ...

```

---



```

0      0      0      0      Y75  Y76  Y77  0; ...
0      Y82  0      0      0      Y86  0      Y88;];

Io = det(t11osV7)/(RC2*(det(t11nim)));
t11 = Io;

% EVALUATION OF THE FEEDBACK PARAMETER t22
t22 = -T/t12;

% EVALUATION OF THE FEEDBACK PARAMETER Beta
Beta = t22/t21;

% The total gain composed of feedback parameters does not
% show the change of resonance frequency
% due to the evaluation of loop gain, which breaks the effect of gain stage Q1
AOL = t11 + t12*t21;
AFF = (t11 + t12*t21)/(1+T);
AF = 20*log10(abs(AFF));

% Calculating Beta using the direct formula
Beta2 = abs(RE2*ZL/( RF*(RE2+ZL+ZC) + ZL*(RE2+ZC) ));

% EVALUATION OF THE FREQUENCY RESPONSE OF THE COMPLETE CIRCUIT
Y11 = 1/RS + 1/RI;
Y12 = Y21 = -1/RI;
Y22 = 1/RI + 1/RF + 1/rpi1;
Y23 = -1/rpi1;
Y28 = Y82 = -1/RF;
Y32 = -(BF1+1)/rpi1;
Y33 = (BF1+1)/rpi1 + 1/RE1;
Y42 = BF1/rpi1;
Y43 = -BF1/rpi1;
Y44 = 1/RC1 + 1/RG1;
Y45 = Y54 = -1/RG1;
Y55 = 1/RG1 + 1/RG2 + 1/rpi2;
Y56 = -1/rpi2;
Y65 = -(BF2+1)/rpi2;
Y66 = (BF2+1)/rpi2 + 1/RE2 + 1/ZC;
Y68 = Y86 = -1/ZC;
Y75 = BF2/rpi2;
Y76 = -BF2/rpi2;
Y77 = 1/RC2;
Y88 = 1/RF + 1/ZL + 1/ZC;

```

---

```

XosV7 = [ Y21  Y22  Y23  0  0  0  Y28;...
         0  Y32  Y33  0  0  0  0; ...
         0  Y42  Y43  Y44  Y45  0  0; ...
         0  0  0  Y54  Y55  Y56  0; ...
         0  0  0  0  Y65  Y66  Y68; ...
         0  0  0  0  Y75  Y76  0; ...
         0  Y82  0  0  0  Y86  Y88;];

XosV4 = [ Y21  Y22  Y23  0  0  0  Y28;...
         0  Y32  Y33  0  0  0  0; ...
         0  Y42  Y43  Y45  0  0  0; ...
         0  0  0  Y55  Y56  0  0; ...
         0  0  0  Y65  Y66  0  Y68; ...
         0  0  0  Y75  Y76  Y77  0; ...
         0  Y82  0  0  Y86  0  Y88;];

Xnim = [ Y11  Y12  0  0  0  0  0  0; ...
        Y21  Y22  Y23  0  0  0  0  Y28;...
        0  Y32  Y33  0  0  0  0  0; ...
        0  Y42  Y43  Y44  Y45  0  0  0; ...
        0  0  0  Y54  Y55  Y56  0  0; ...
        0  0  0  0  Y65  Y66  0  Y68; ...
        0  0  0  0  Y75  Y76  Y77  0; ...
        0  Y82  0  0  0  Y86  0  Y88;];

% output from node 4
I1 = det(XosV4)/(RC1*(det(Xnim)));
% output from node 7
I2 = -det(XosV7)/(RC2*(det(Xnim)));

% magnitudes in decibels
AV1 = 20*log10(abs(I1));
AV2 = 20*log10(abs(I2));

% storing the data into arrays
HH=[HH abs(T)];
KK=[KK AV1];
JJ=[JJ AV2];
LL=[LL abs(t22)];
X = [X f];
endif;

% determine the next frequency according to the chosen mode
if (mode == 1)

```

```
f = f + adder*10^j;
else
f = f*multiplr;
endif;
end; % while
end; % for
% store data to be saved and/or plotted
PLOTBUF = [PLOTBUF X' HH'];
%save -ascii T10.data PLOTBUF

% Print the value of T0
T0

% create plots
figure(1);
semilogx(X,HH)
figure(2);
semilogx(X,KK)
```

---