
BASIC FORMS OF FEEDBACK IN VOLTAGE AND CURRENT
AMPLIFIERS

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1 INTRODUCTION

Feedback is a fundamental concept in all fields of engineering. In electronics negative feedback is used for stabilising gain, modifying impedance, extending bandwidth and reducing noise. References to the terms of DC and AC feedback imply that the feedback stabilises and enhances either DC or AC behaviour of the circuit. Typically the feedback is a mixture of AC and DC feedback. The most important parameters to describe feedback are the return ratio T and the feedback factor β . The following discussion and analysis aims to explain how these parameters are evaluated and what do they actually mean in practice.

Before getting down to business, it is worth mentioning a good tool for analysing feedback in electronics and in general: the signal flow graph. Figure 1 introduces the most basic elements used for visualising a complete signal chain as a flow graph. The key point is to understand how the mul-

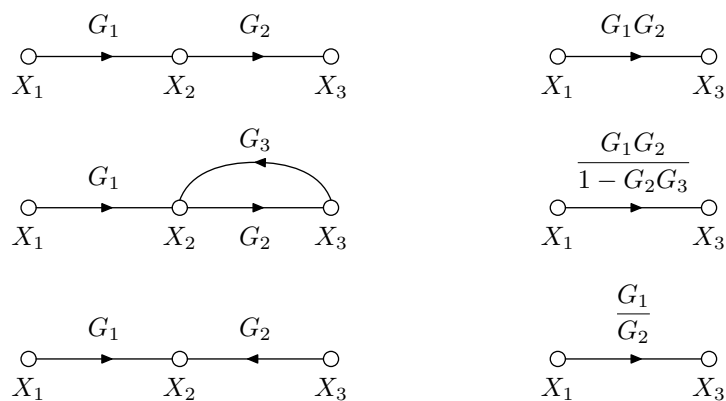


Figure 1: Basic forms of signal flow functions and their 'simplifications'

tipliers G_x between two junctions are related to each other when the flow graph is simplified by reducing the number of branches. The mathematical 'proof' of these multiplier combinations is simple; for example the bottom signal flow is evaluated as

$$\begin{aligned} X_2 &= G_1 X_1 \\ X_2 &= G_2 X_3 \quad \rightarrow \quad X_3 = \frac{G_1}{G_2} X_1. \end{aligned}$$

The multiplier is applied to the junction from where the arrow leaves and the result of this multiplication is the junction where the arrow points to.

A general feedback cycle is easily demonstrated with the signal flow graph of Figure 2. The form of presentation is adapted from the textbook of Millman

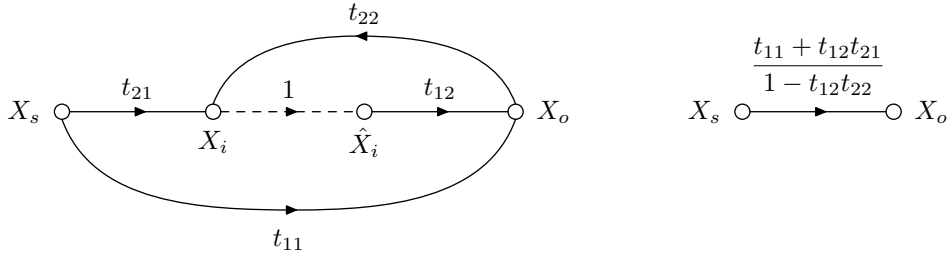


Figure 2: Signal flow graph to explain feedback

(Microelectronics 2nd ed.). The idea is to introduce a new set of 'two-port' parameters $t_{11}, t_{12}, t_{21}, t_{22}$, which act as multipliers between the branches of the feedback circuit. The method of evaluating the multipliers is to utilise the idea of superposition. When evaluating each multiplier one by one, some of the junctions (voltage or current sources) are given a zero value to isolate a specific section of the circuit. The dashed line between X_i and \hat{X}_i should be interpreted as if the connection between these two points had been broken. Since \hat{X}_i is just a parameter related to the controlled source, the connection between X_i and \hat{X}_i in the small signal model can be directly considered as 'broken' and the circuit configuration does not need any changes for calculations. Detailed examples of this method are to follow later on.

The simplified equation for the feedback signal flow is derived by writing the equations

$$X_i = t_{21}X_s + t_{22}X_o \quad (1)$$

$$\hat{X}_i = X_i \quad (2)$$

$$X_o = t_{12}\hat{X}_i + t_{11}X_s \quad (3)$$

and then simply solving the relation between the input X_s and the output X_o . When comparing to the "general equation" of feedback,

$$X_o = \frac{t_{11} + t_{12}t_{21}}{1 - t_{12}t_{22}}X_s = \frac{A_{OL}}{1 + T}X_s, \quad (4)$$

each of the parameters are given their special meaning. The mathematical definitions for the parameters are stated as:

$$\begin{aligned} t_{11} &= \left. \frac{X_o}{X_s} \right|_{\hat{X}_i=0} & t_{12} &= \left. \frac{X_o}{\hat{X}_i} \right|_{X_s=0} \\ t_{21} &= \left. \frac{X_i}{X_s} \right|_{X_o=0} & t_{22} &= \left. \frac{X_i}{X_o} \right|_{X_s=0} \\ -T &= \left. \frac{X_i}{\hat{X}_i} \right|_{X_s=0} = t_{12}t_{22} = t_{12}t_{21} \frac{t_{22}}{t_{21}} \approx A_{OL}\beta \end{aligned}$$

On the right side of each definition is an indication which source should be set to zero when evaluating the parameter. The source can be current or voltage depending on the feedback configuration. Often the trickiest part is to handle the output source correctly when evaluating t_{21} . It helps to think that when setting the output source to zero, the output signal should be isolated so that no signal goes to the feedback loop. When considering parameters t_{12} and t_{21} , the feedback part of the circuit is typically seen as a load and therefore is a natural part of these parameters. The return ratio T is a very useful parameter, which is especially needed when evaluating the impedance changes due to the feedback configuration.

Typically circuits with feedback loops are analysed by extracting the amplifier gain path and the feedback path as separate parameters. By the rules indicated in figure 1, the gain path forward transmission transfer function is

$$\frac{X_o}{X_s} = t_{12}t_{21}. \quad (5)$$

If the feedforward path t_{11} is neglected (as is typically safe to do), then the so called open loop gain $A_{OL} = t_{12}t_{21}$. With the same reasoning the feedback path reverse transmission transfer function is

$$\frac{X_s}{X_o} = \frac{t_{22}}{t_{21}} = \beta. \quad (6)$$

So the general idea is to evaluate a transfer function between input and output to both directions of signal flow and indicate them as A_{OL} and β . The most interesting feedback parameter is the *feedback factor* β , which describes the reverse transmission of the feedback network. Quite often approximate feedback analysis methods give a false meaning to β , as if it would be a factor related to t_{22} path only. Generally β describes the reverse transfer function where the output node X_o is taken as the signal source and the source node X_s is treated as the end point of the feedback signal.

The input and output impedances of a circuit with feedback can be evaluated using the Blackman's impedance formula

$$Z_F = Z_D \frac{1 + T_{SC}}{1 + T_{OC}}, \quad (7)$$

where Z_D is the input or output impedance evaluated from a 'dead system' (gain is set to zero) and T_{SC}, T_{OC} are the return ratio T evaluated in short-circuited (SC) and open-circuited (OC) conditions respectively. Typically it is enough to evaluate T once according to its definition and then in the obtained equation set values 0 (short) or ∞ (open) to the input and terminating

impedances (R_s and R_L). The 'dead system' is obtained by suppressing the controlled source ($\hat{X}_i = 0$) just like it is done when evaluating the t_{11} feed-forward parameter. The 'dead system' input and output impedances Z_{ID} and Z_{OD} are then evaluated from the zero-gain system. These impedances are scaled with the return ratio depending on the feedback configuration used. Concrete examples will follow later on.

The following little snippet demonstrates the effect of two different feedback configurations, which are applied to the same direct-coupled pair BJT amplifier setup. Negative feedback is used mostly for making the amplifier more stable with respect to some critical parameter, mainly current or voltage gain and enhancing the input and output impedances of any gain stage within a circuit.

The analysed circuit is a BJT direct-coupled pair circuit as shown in Figure 3. This basic form of the circuit does not contain any global feedback loops. In the upcoming analysis the circuit is treated as a current amplifier by adding a current-shunt feedback loop and as a voltage amplifier by adding a voltage-series feedback loop.

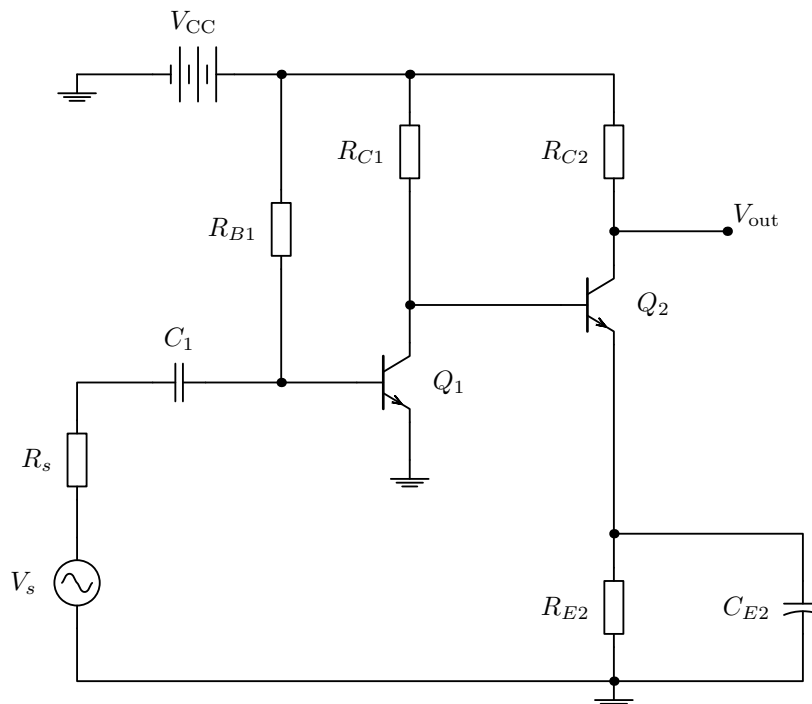


Figure 3: A bipolar junction transistor direct-coupled pair amplifier

2 CURRENT-SHUNT FEEDBACK

The current-shunt feedback is typically applied in current amplifiers, where the output current is sampled and fed back to the input in a parallel (shunt) connection. This feedback configuration is also known as the 'shunt-series' type because the output current is probed by a series connection (parallel probing of current is not possible). In the feedback analysis the amplifier element is modelled as a general 'two-port' system (see Figure 4), from where the series and shunt connections are easily distinguished.

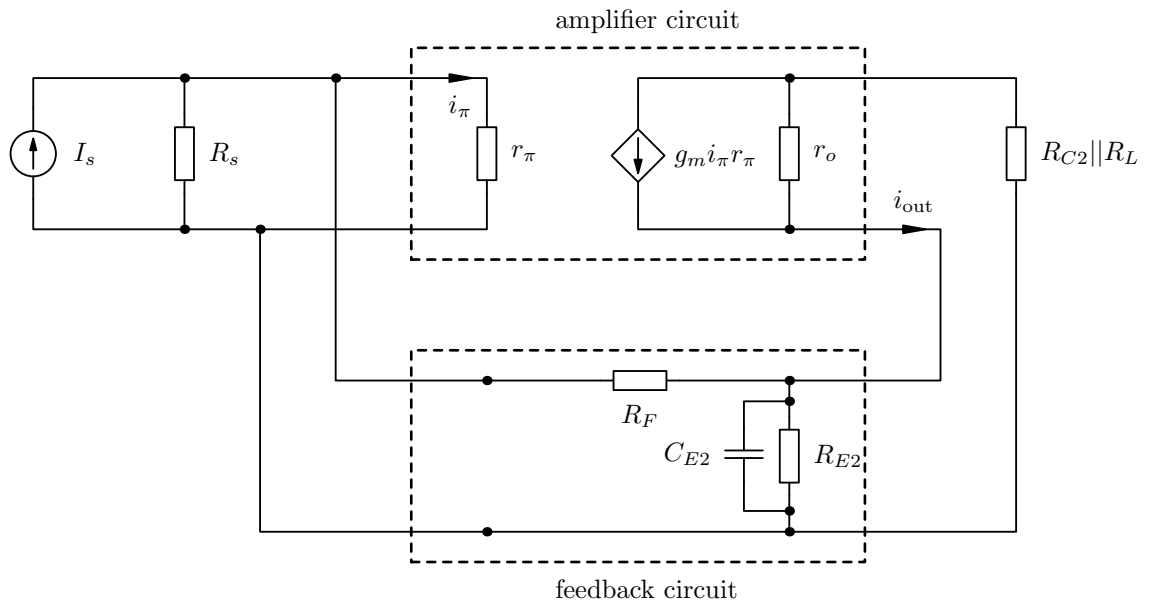


Figure 4: A current amplifier with shunt feedback

The feedback network in Figure 4 is implemented as a simple passive current divider, where the output current is divided between resistors R_F and R_{E2} . The capacitor C_{E2} acts as a short circuit for AC currents so that the feedback loop in this case is closed for DC current only. The feedback configuration is drawn specifically this accurately to relate to the circuit under analysis.

An example amplifier structure using the current-shunt configuration is shown in Figure 5. This configuration is chosen because it is used as a basis in the early 'fuzz-face' and 'wah-wah' guitar effect devices. The feedback loop is implemented by connecting the emitter of the second amplifier stage to the base of the first amplifier stage via resistor R_F . As already mentioned, the feedback loop is closed only for direct current. Therefore, the feedback is used here mainly to enhance the biasing stability of the amplifier. To see

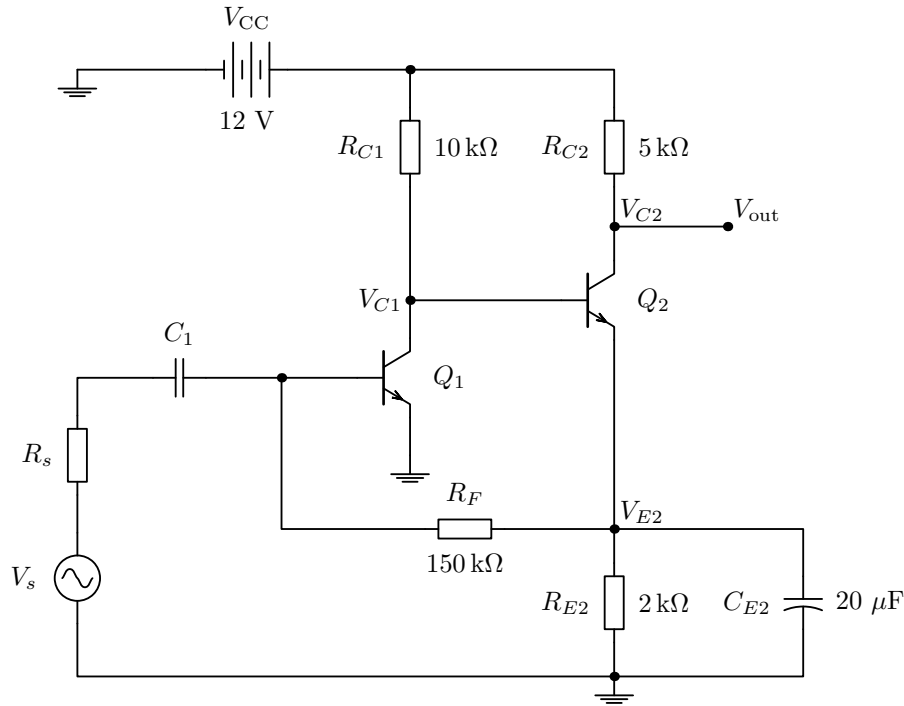


Figure 5: A direct-coupled pair amplifier with current-shunt feedback

the effects of the feedback loop, the amplifier should be given an in-depth analysis from the viewpoint of DC and AC operation.

2.1 DC analysis

The DC model is redrawn for analysis purposes in Figure 6. Clearly the feedback loop is used for giving the needed base current for transistor Q_1 .

A reasonably systematic method for the DC bias analysis is to examine the independent voltage nodes of the circuit. The direct current model in Figure 6 has two distinct voltage nodes, labelled as V_{C1} and V_{E2} . It is also beneficial to notice that $V_{E2} = V_{C1} - V_{BE2}$. This reduces the number of distinct voltage nodes to one.

Based on Kirchhoff's current law, the sum of currents entering a node equals the sum of currents leaving a node. Therefore, the current equations for the voltage nodes V_{E2} and V_{C1} are:

$$I'_{E2} = I_{E2} - I_{B1} = (\beta_{F2} + 1)I_{B2} - I_{B1}$$

$$I'_{C1} = I_{C1} + I_{B2} = \beta_{F1}I_{B1} + I_{B2}.$$

These equations can be combined to eliminate the base current I_{B2} and then

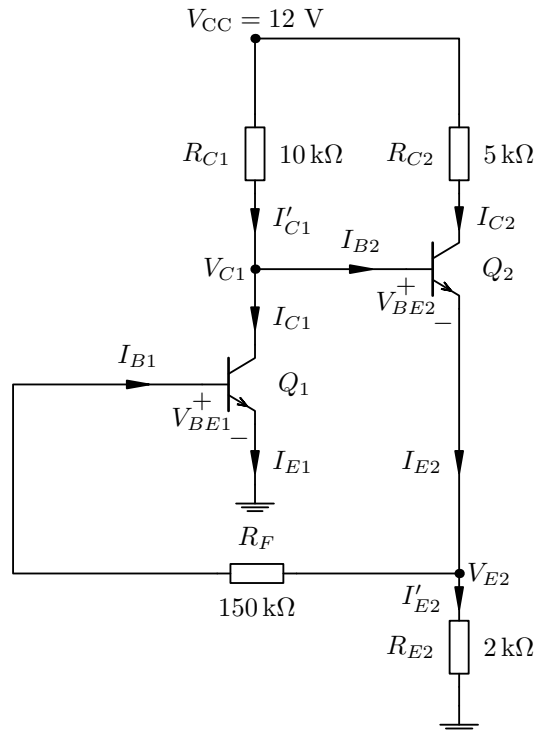


Figure 6: A DC model of the amplifier with current-shunt feedback

the current equation reads

$$I'_{C1} = \beta_{F1} I_{B1} + \frac{I'_{E2}}{\beta_{F2} + 1} + \frac{I_{B1}}{\beta_{F2} + 1}.$$

These currents can be expressed with the node voltages as:

$$I'_{E2} = \frac{V_{E2}}{R_{E2}} \quad ; \quad I'_{C1} = \frac{V_{CC} - V_{C1}}{R_{C1}} \quad ; \quad I_{B1} = \frac{V_{E2} - V_{BE1}}{R_F}$$

and after substituting the voltage equations,

$$\frac{V_{CC} - V_{C1}}{R_{C1}} = \beta_{F1} \frac{V_{E2} - V_{BE1}}{R_F} + \frac{V_{E2}}{R_{E2}(\beta_{F2} + 1)} + \frac{V_{E2} - V_{BE1}}{R_F(\beta_{F2} + 1)}.$$

With the fact that $V_{E2} = V_{C1} - V_{BE2}$, one has an equation from where the voltage V_{C1} can be solved. Based on this analysis, all the other biasing voltages and currents can also be solved.

2.2 AC analysis

Next the focus moves to the AC feedback analysis, which for the given circuit is simple: there is no AC feedback because of capacitor C_{E2} . But since it would be nice to introduce the tools for feedback analysis, let's remove all the capacitors from the circuit and analyse the feedback properties of the circuit at mid-frequency range.

Considering the frequency response (AC) analysis, it is necessary to redraw the circuit using the small-signal model of the BJT. A low-frequency BJT model is sufficient in this case because the focus is on the audio frequency analysis only. The small-signal model in Figure 7 still has the capacitors drawn in place, but the following analysis assumes those are removed from the circuit.

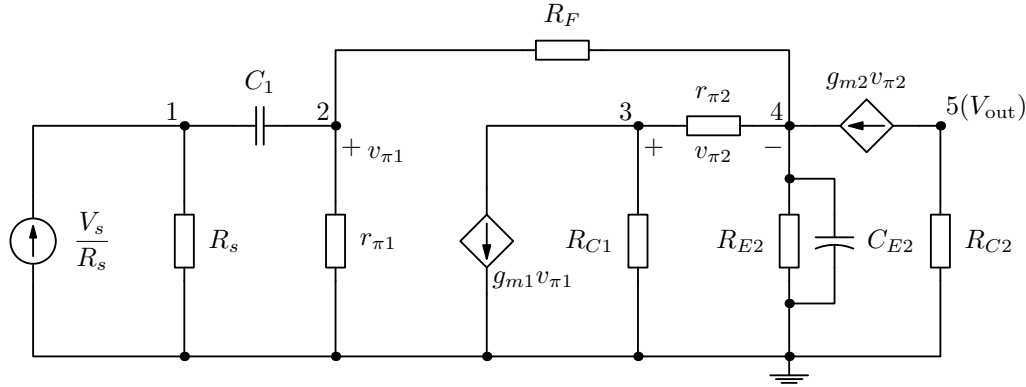


Figure 7: A small-signal model of the direct-coupled pair amplifier with current-shunt feedback

Let's apply the general feedback analysis to this circuit and find equations for t_{12} , t_{21} , t_{22} , T and β . Since the feedback signal is current, it is convenient that the controlled sources are transformed into 'current mode' using the expansion

$$g_m v_\pi = g_m r_\pi \frac{v_\pi}{r_\pi} = \beta_F i_\pi. \quad (8)$$

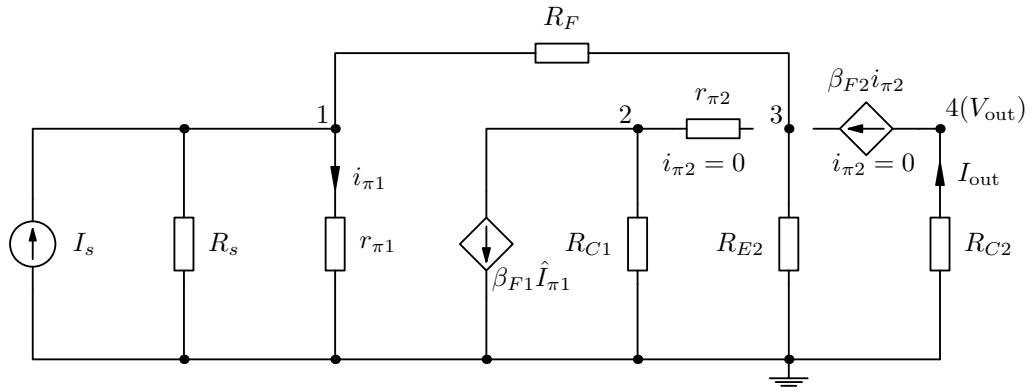
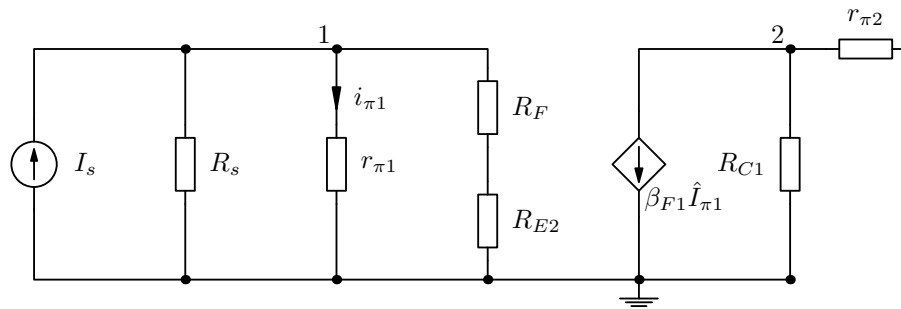
When following this feedback analysis it is necessary that one controlled source be chosen as an independent source. This independent source is identified later on by using the notation

$$\beta_F \hat{I}_\pi. \quad (9)$$

2.3 t_{21} feedback parameter

The return ratio T and t_{12} are obtained from the same circuit model where $X_s = I_s$ is set to zero. To evaluate t_{21} , a circuit with the output source $X_o = I_o = 0$ needs to be drawn. This circuit is shown in Figure 8 and simplified further in Figure 9.

The matrix equation describing the circuit in Figure 9 is given by equation (10).

Figure 8: Circuit model for evaluating the t_{21} feedback parameterFigure 9: Simplified circuit model for evaluating the t_{21} feedback parameter

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} + \frac{1}{R_F + R_{E2}} & 0 \\ 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_s \\ -\beta_{F1} \hat{I}_{\pi 1} \end{bmatrix} \quad (10)$$

This matrix equation can be written directly by inspecting the circuit model and using the rules of the nodal analysis. The nodal analysis is used, because the second controlled current source cannot be transformed as a voltage source due to a missing parallel resistance. According to the rules of the nodal analysis, the admittance terms are collected to the main matrix and the current source terms are placed into the current vector. The voltage nodes in the voltage vector are the variables to be solved from this equation.

For evaluation purposes it is convenient that all controlled source terms from the current vector are moved to the admittance matrix. Because the matrix equation is just a set of linear equations, the controlled source terms are subtracted or added to both sides of the equation. It is also necessary to identify to which voltage node variables the controlled source term relates to. Based on this discussion, the matrix equation (11) is readily modified in

the desired form.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} + \frac{1}{R_F + R_{E2}} & 0 \\ \frac{\beta_{F1}}{r_{\pi 1}} & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \end{bmatrix} \quad (11)$$

The feedback parameter t_{21} was defined as the fraction $\frac{X_i}{X_s}$, where the source terms in this current amplifier case are defined as currents. Therefore, the resulting equation for t_{21} is

$$t_{21} = \frac{i_{\pi 1}}{I_s} = \frac{V_1}{r_{\pi 1} I_s} = \frac{R_s(R_{E2} + R_F)}{(r_{\pi 1} + R_s)(R_{E2} + R_F) + R_s r_{\pi 1}}. \quad (12)$$

As a single parameter this does not yet describe much of the circuit properties, it is just a multiplier between the input source and the input of the amplifier.

2.4 Return ratio T and t_{12} feedback parameter

The circuit model needs to be redrawn for analysing the return ratio T and the feedback parameter t_{12} . Figure 10 shows the correct model, where the input source is removed due to the rule $X_s = I_s = 0$. When the value of a current source is set to 0, it describes an open-circuit condition and the branch containing the current source is simply removed from the circuit as seen from Figure 10.

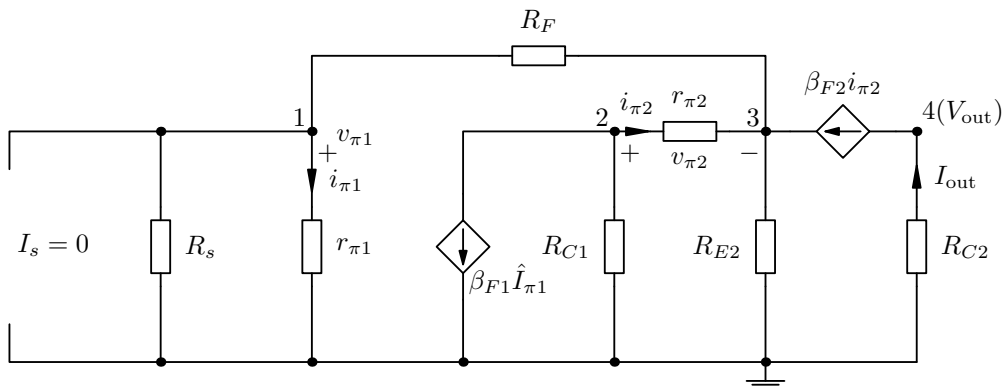


Figure 10: Circuit model for evaluating feedback parameters T and t_{12}

The matrix equation describing the circuit model of Figure 10 is written as

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} & 0 \\ 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & -\frac{1}{r_{\pi 2}} & 0 \\ -\frac{1}{R_F} & -\frac{1}{r_{\pi 2}} & \frac{1}{R_F} + \frac{1}{R_{E2}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & 0 & 0 & \frac{1}{R_{C2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -\beta_{F1} \hat{I}_{\pi 1} \\ \beta_{F2} i_{\pi 2} \\ -\beta_{F2} i_{\pi 2} \end{bmatrix}.$$

This equation needs to be modified so that it is easy to solve the fractions $-\frac{i_{\pi 1}}{\hat{I}_{\pi 1}}$ and $\frac{I_{out}}{\hat{I}_{\pi 1}}$ as required by the definitions of the feedback parameters. Therefore the independent controlled source term $\hat{I}_{\pi 1}$ is left to the current vector, while all other current source terms are moved to the admittance matrix. The resulting matrix equation is of the form shown below and the unknown voltages can be relatively easily solved from this matrix equation using the Cramer's rule.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} & 0 \\ 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & -\frac{1}{r_{\pi 2}} & 0 \\ -\frac{1}{R_F} & -\frac{\beta_{F2} + 1}{r_{\pi 2}} & \frac{1}{R_F} + \frac{1}{R_{E2}} + \frac{\beta_{F2} + 1}{r_{\pi 2}} & 0 \\ 0 & \frac{\beta_{F2}}{r_{\pi 2}} & -\frac{\beta_{F2}}{r_{\pi 2}} & \frac{1}{R_{C2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -\beta_{F1} \hat{I}_{\pi 1} \\ 0 \\ 0 \end{bmatrix}.$$

After solving the fraction $-\frac{i_{\pi 1}}{\hat{I}_{\pi 1}}$ one has the equation for the return ratio

$$T = -\frac{i_{\pi 1}}{\hat{I}_{\pi 1}} = \frac{R_P}{r_{\pi 1}} \frac{\beta_{F1} R_{C1} (\beta_{F2} + 1) R_{E2}}{(R_{E2} + R_F + R_P)(R_{C1} + r_{\pi 2}) + (\beta_{F2} + 1) R_{E2} (R_F + R_P)},$$

where a parallel resistance term $R_P = R_s || r_{\pi 1}$ has been taken into use to simplify the equations. In a similar fashion the feedback parameter t_{12} is solved using the same model, and the result is

$$t_{12} = \frac{V_4}{R_{C2} \hat{I}_{\pi 1}} = -\frac{\beta_{F1} R_{C1} \beta_{F2} (R_{E2} + R_F + R_P)}{(R_{E2} + R_F + R_P)(R_{C1} + r_{\pi 2}) + (\beta_{F2} + 1) R_{E2} (R_F + R_P)}.$$

2.5 t_{22} , feedback factor β and gain factor K

If the approximation is made that $\beta_{F2} + 1 \approx \beta_{F2}$, then the multiplier t_{22} will simplify considerably to the form

$$t_{22} = -\frac{T}{t_{12}} = \frac{R_S R_{E2}}{(r_{\pi 1} + R_S)(R_F + R_{E2}) + R_S r_{\pi 1}},$$

and then the equation for the feedback factor becomes

$$\beta = \frac{t_{22}}{t_{21}} = \frac{R_{E2}}{R_{E2} + R_F}. \quad (13)$$

The gain factor K is the negative inverse of the feedback factor:

$$K = -\frac{1}{\beta} = -\frac{t_{21}}{t_{22}} = -\frac{R_{E2} + R_F}{R_{E2}}, \quad (14)$$

And since all the feedback parameters have been evaluated as currents, the gain factor represents current gain. If the return ratio T is large, then K is quite accurate approximation of the total gain of the circuit. For better accuracy The total gain of the circuit can be evaluated using the general formula (4) as

$$A_i = \frac{t_{11} + KT}{1 + T}, \quad (15)$$

which needs the t_{11} parameter to get the exact value, but its effect to the final result is small so it can be left out.

2.6 Input and output impedance

For evaluating the input impedance one must draw a circuit model for the so called 'dead-system'. The dead-system impedance was needed for using the Blackman's impedance formula (7). Figure 11 shows the dead-system model with the independent controlled source suppressed. Figure 11 includes also an ideal test current source I_T , and related internal impedance R_s . However, since this is a test source, it should be ideal. If the test source would be an ideal voltage source, then $R_s = 0$, but now in the current source $R_s = \infty$ and can be considered disconnected from the circuit. With the test source included, the input impedance is determined by sourcing current and measuring the resulting voltage at voltage node V_1 .

The impedance is evaluated by solving V_1 from the matrix equation below and dividing by the test current I_T . Notice that the matrix does not include

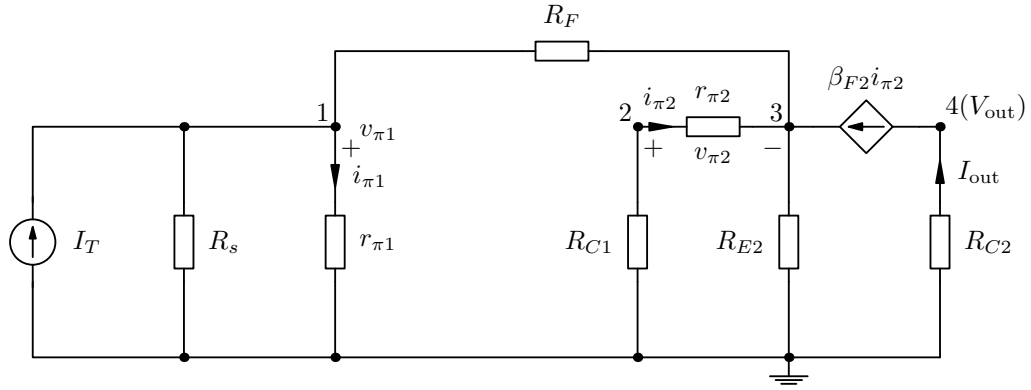


Figure 11: Circuit model for evaluating the dead-system input impedance

the admittance of R_s , since its value is zero in the analysis.

$$\begin{bmatrix} \frac{1}{r_{\pi 1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} & 0 \\ 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & -\frac{1}{r_{\pi 2}} & 0 \\ -\frac{1}{R_F} & -\frac{\beta_{F2} + 1}{r_{\pi 2}} & \frac{1}{R_F} + \frac{1}{R_{E2}} + \frac{\beta_{F2} + 1}{r_{\pi 2}} & 0 \\ 0 & \frac{\beta_{F2}}{r_{\pi 2}} & -\frac{\beta_{F2}}{r_{\pi 2}} & \frac{1}{R_{C2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_T \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

As a side note, for getting a decent expression for the dead-system input impedance it is not necessary to solve the matrix equation, because a direct inspection of Figure 11 shows the input impedance to be approximately the parallel connection of the branches $r_{\pi 1}$ and $R_F + R_{E2}$. However, for comparison, the dead-system input impedance solved from the matrix equation is

$$Z_{ID} = \frac{r_{\pi 1} R_F [R_{E2} (\beta_{F2} + 1) + r_{\pi 2} + R_{C1}] + r_{\pi 1} R_{E2} (r_{\pi 2} + R_{C1})}{(r_{\pi 1} + R_F) [R_{E2} (\beta_{F2} + 1) + r_{\pi 2} + R_{C1}] + R_{E2} (r_{\pi 2} + R_{C1})}$$

The Blackman's formula requires also the return ratio to be evaluated in short-circuit and open-circuit conditions. Basically this is simple, since the methodology states that R_s and R_L are not part of the amplifier feedback cycle when considering the input and output connections. Therefore, one can get the short-circuit and open-circuit conditions by inspection from the already evaluated expression for T . Setting $R_s = 0$ in the return ratio of the current-shunt feedback system creates the short-circuit condition. For this condition $T(R_s \rightarrow 0) = 0$, so the only meaningful value for the return ratio

must come from setting $R_s = \infty$. Hence,

$$T(R_s \rightarrow \infty) = \frac{\beta_{F1} R_{C1} (\beta_{F2} + 1) R_{E2}}{(R_{E2} + R_F + r_{\pi1})(R_{C1} + r_{\pi2}) + (\beta_{F2} + 1) R_{E2} (R_F + r_{\pi1})},$$

and the use of the Blackman's impedance formula for the input impedance with feedback gives

$$Z_{IF} = \frac{Z_{ID}}{1 + T(R_s \rightarrow \infty)} \approx \frac{r_{\pi1} || (R_F + R_{E2})}{1 + T(R_s \rightarrow \infty)},$$

where the approximated expression for the dead-system input impedance has been used. So the current-shunt feedback reduces the input impedance by the amount determined by the open-circuited return ratio.

The output impedance evaluation needs a bit of thought. The terminating impedance is seen as $\frac{R_{C2}}{R_{C2}}$ in the loop gain equation and basically cancels itself out. Although setting $R_{C2} \rightarrow 0$ would be OK (results into 1), one cannot set $R_{C2} \rightarrow \infty$ in the loop gain equation (infinity divided by infinity is undefined).

So to evaluate T in the open circuit state, a new small-signal model should be drawn. Here we will benefit from knowing that the feedback loop in the output is in series with the load resistance R_{C2} . If R_{C2} is taken out from the circuit the output current becomes 0 and no current is flowing in the feedback loop. Therefore the small-signal model is drawn much like in Figure 8 and the loop gain $T(R_{C2} \rightarrow \infty)$ is 0, because the loop is broken.

So eventually it is evident that the output impedance is evaluated from the equation

$$Z_{OF} = Z_{OD}(1 + T(R_{C2} \rightarrow 0)), \quad (16)$$

where $T(R_{C2} \rightarrow 0) = T$ and $Z_{OD} = \infty$, since the intrinsic transistor resistance r_0 in this simplified model was approximated to infinity and neglected from the analysis. The output resistance for the whole circuit is therefore R_{C2} .

2.7 Circuit with R_{E1} included

Some adaptations of this circuit add one extra resistor to the emitter of Q_1 . A good example of using this approach is the Crybaby-style wah-pedal. The obtained results for the circuit without R_{E1} can be extended to cover the circuit with R_{E1} included using a neat little trick. Beware that this trick does not apply for every circuit, but quite often it works nicely. When one

has evaluated T , K and t_{11} for the circuit without R_{E1} , the circuit with R_{E1} included can be modelled by making the following changes:

1. If the expression for T has the combination $g_m r_{\pi 1}$ in the numerator, assign that as β_{F1} .
2. Then assign $r_{\pi 1} = r_{\pi 1} + (\beta_{F1} + 1)R_{E1}$ for all the remaining $r_{\pi 1}$ in the equations of t_{11} , T and K , after this one gets the accurate voltage gain results from equation (15).

Note that this same approach should work for the impedance evaluation as well, but I have not verified that numerically. Just make sure that the combinations of $g_m r_{\pi 1}$ in the numerators of T are identified and marked as β_{F1} before making the adjustment to $r_{\pi 1}$ in all equations.

3 VOLTAGE-SERIES FEEDBACK

As its name suggests, the voltage-series feedback is typically applied in voltage amplifiers. In this feedback type the output voltage is sampled and fed back to the input in series with the signal source. This feedback configuration is also known as the 'shunt-series' type because the output voltage is probed by a shunt connection (probing of voltage with a series connection is not possible). In the feedback analysis the amplifier element and the feedback network are modelled as a general 'two-port' systems (see Figure 12), from where the series and shunt connections are easily distinguished.

An example amplifier structure using the voltage-series configuration is shown in Figure 13. The feedback loop is implemented by connecting the collector of the second amplifier stage to the emitter of the first amplifier stage via resistor R_F .

To see the effects of the feedback loop, the amplifier should be analysed from the viewpoint of DC and AC operation.

3.1 DC analysis

The DC model is redrawn for analysis purposes in Figure 14. The analysis assumes that both of the transistors are biased in the forward-active region. The biasing is however sensitive to the current levels and either of the transistors is easily saturated in this connection. In this case, the feedback loop

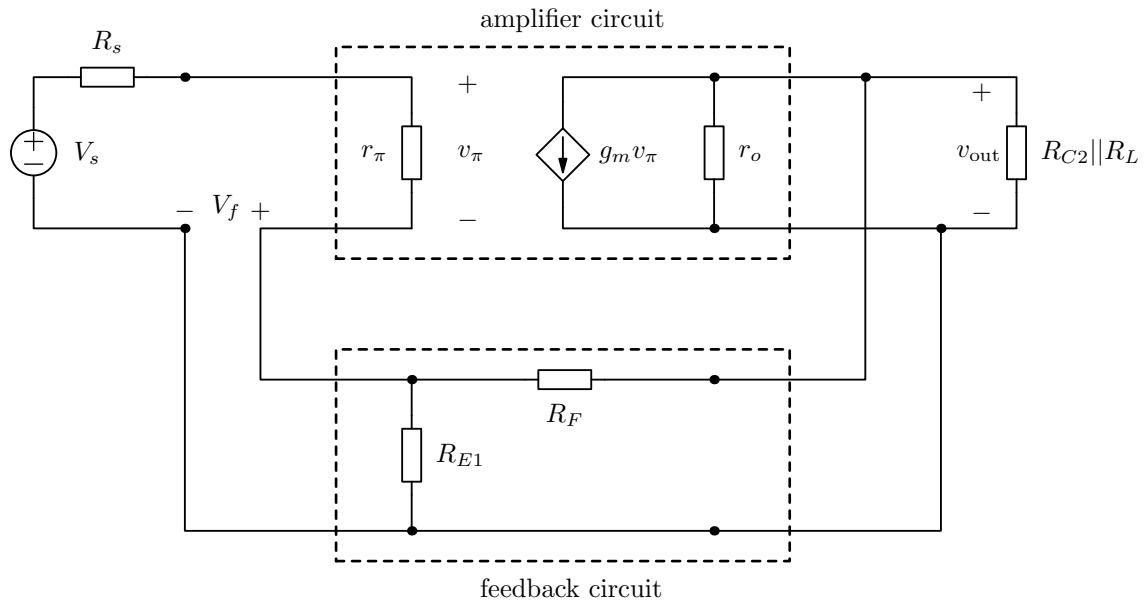


Figure 12: A voltage amplifier with series feedback

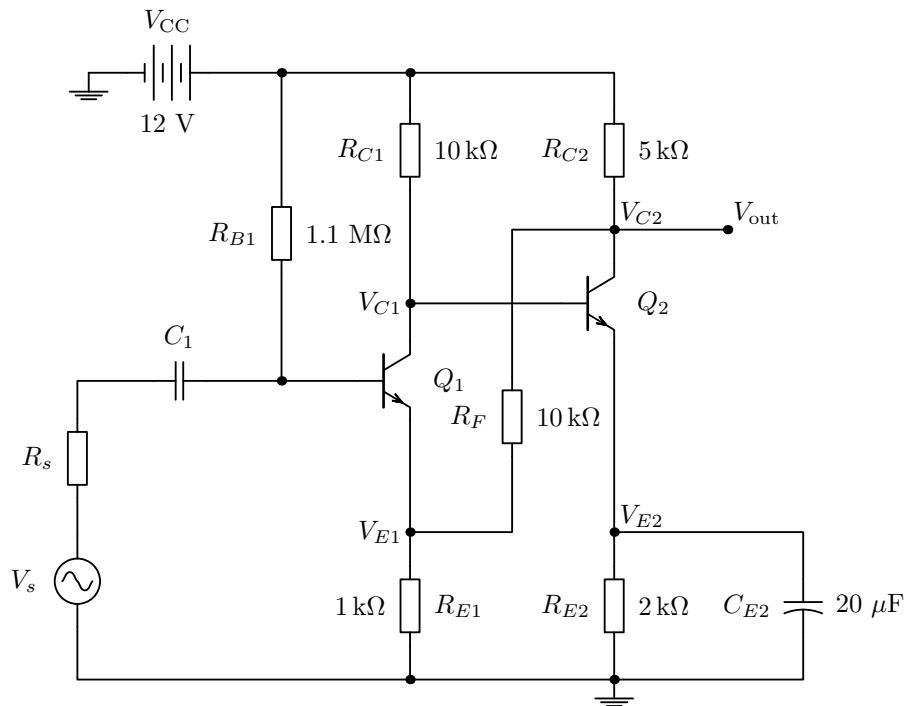


Figure 13: A direct-coupled pair amplifier with voltage-series feedback

affects very little to the DC biasing, because the feedback only affects the potential V_{E1} and does not interfere with any significant transistor bias current.

The circuit in Figure 6 has three distinct voltage nodes, labelled as V_{C1} , V_{E1} and V_{C2} . Based on Kirchhoff's current law, the current equations for the

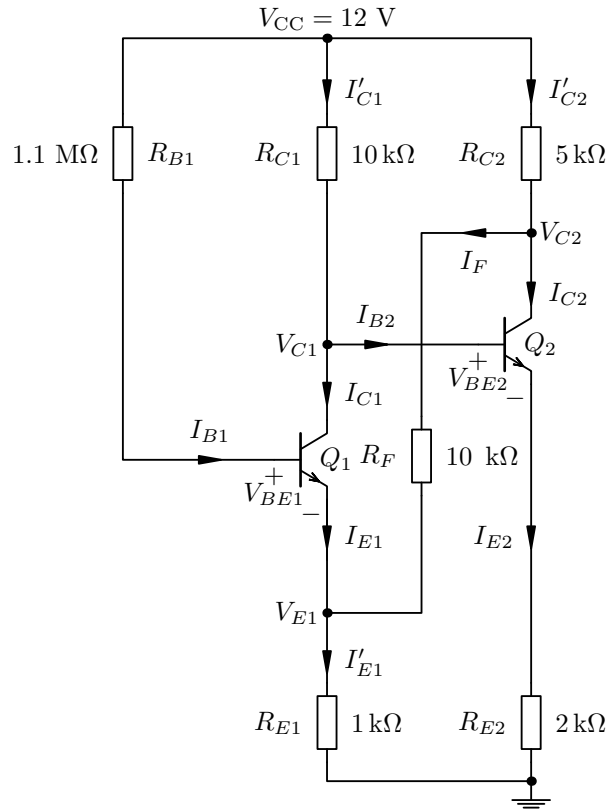


Figure 14: A DC model of the amplifier with voltage-series feedback

nodes are:

$$I'_{E1} = I_{E1} + I_F = (\beta_{F1} + 1)I_{B1} + I_F$$

$$I'_{C1} = I_{C1} + I_{B2} = \beta_{F1}I_{B1} + I_{B2}$$

$$I'_{C2} = I_{C2} + I_F = \beta_{F2}I_{B2} + I_F$$

The essential currents can be expressed with the node voltages as:

$$I'_{E1} = \frac{V_{E1}}{R_{E1}} \quad ; \quad I'_{C1} = \frac{V_{CC} - V_{C1}}{R_{C1}} \quad ; \quad I_{B1} = \frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}}$$

$$I_F = \frac{V_{C2} - V_{E1}}{R_F} \quad ; \quad I'_{C2} = \frac{V_{CC} - V_{C2}}{R_{C2}} \quad ; \quad I_{B2} = \frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2} + 1)}$$

Substitutions of the voltage equations into the current equations leads to the following three equations

$$\frac{V_{E1}}{R_{E1}} = (\beta_{F1} + 1) \frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}} + \frac{V_{C2} - V_{E1}}{R_F}$$

$$\frac{V_{CC} - V_{C1}}{R_{C1}} = \beta_{F1} \frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}} + \frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2} + 1)}$$

$$\frac{V_{CC} - V_{C2}}{R_{C2}} = \frac{\beta_{F2}}{\beta_{F2} + 1} \frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2} + 1)} + \frac{V_{C2} - V_{E1}}{R_F}$$

These equations can be organised into a matrix equation, from where each of the node voltages can be solved using the Cramer's rule. The matrix equation is:

$$\begin{bmatrix} \frac{1}{R_{E1}} + \frac{\beta_{F1} + 1}{R_{B1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ -\frac{\beta_{F1}}{R_{B1}} & \frac{1}{R_{C1}} + \frac{1}{R_{E2}(\beta_{F2} + 1)} & 0 \\ -\frac{1}{R_F} & \frac{\beta_{F2}}{\beta_{F2} + 1} \frac{1}{R_{E2}} & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_{E1} \\ V_{C1} \\ V_{C2} \end{bmatrix} = \begin{bmatrix} (\beta_{F1} + 1) \frac{V_{CC} - V_{BE1}}{R_{B1}} \\ \frac{V_{CC}}{R_{C1}} + \beta_{F1} \frac{V_{BE1} - V_{CC}}{R_{B1}} + \frac{V_{BE2}}{R_{E2}(\beta_{F2} + 1)} \\ \frac{V_{CC}}{R_{C1}} + \frac{\beta_{F2}}{\beta_{F2} + 1} \frac{V_{BE2}}{R_{E2}} \end{bmatrix}$$

Based on this analysis, all the other biasing voltages and currents can be solved using the magnitudes of the node voltages.

3.2 AC analysis

Then one proceeds towards the AC analysis. Figure 15 shows the complete small-signal model of the system, but to ease out the calculations, the simplified model of Figure 16 is adequate. The simplifications concern the removal of DC-blocking capacitors and the high-valued biasing resistor R_{B1} .

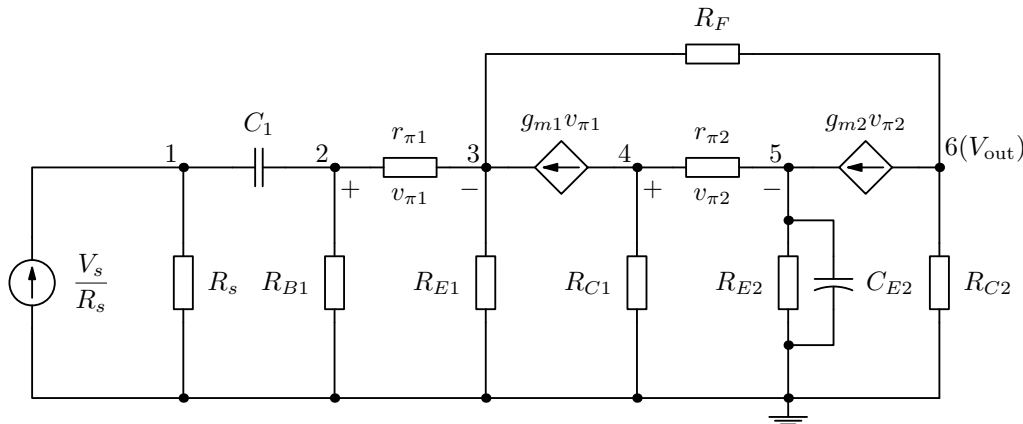


Figure 15: A small-signal model of the direct-coupled pair amplifier with voltage-series feedback

The analysis approach for the voltage-series case is the same as for the current-shunt feedback configuration, but now we are aiming to perfection and calculate also the feedback parameter t_{11} , which is often neglected, because of its small significance in the expression for the total gain of the feedback system.

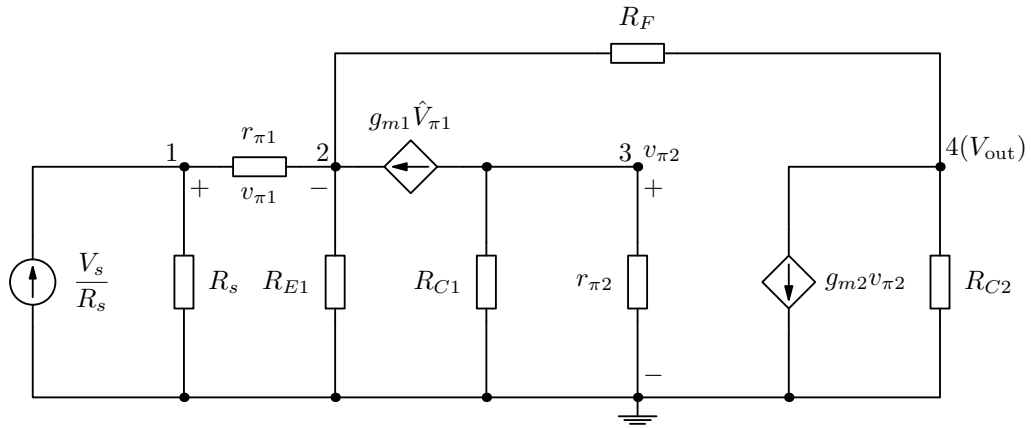
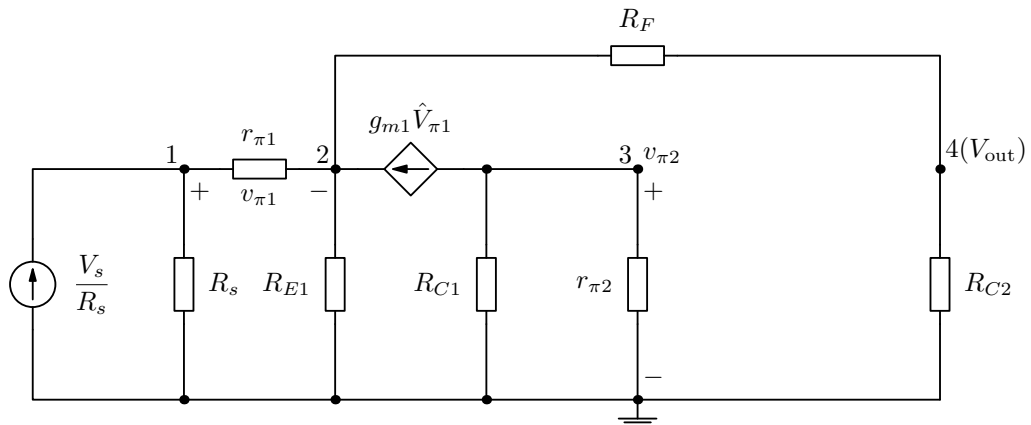


Figure 16: A simplified model for the mid-band frequencies

3.3 t_{11} feedback parameter

So first we evaluate the dead-system gain t_{11} from the circuit of Figure 17, where the second controlled source Q_2 has been suppressed as the theory requires.

Figure 17: Small-signal model for evaluating the feedback parameter t_{11}

However, when calculating the other feedback parameters, we are using Q_1 as the selected controlled source. This is confusing, but this has to be done to get exact results for this circuit. The reason is the local feedback loop around Q_1 , which has to be retained for the dead-system gain evaluation. The obtained results have been verified against direct evaluation of the complete transfer function and it validates that the local feedback loop is part of the dead-system circuit.

Anyway, the matrix equation to evaluate t_{11} from Figure 17 is

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{1}{r_{\pi 1}} & \frac{1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ 0 & 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & 0 & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ g_{m1} \hat{V}_{\pi 1} \\ -g_{m1} \hat{V}_{\pi 1} \\ 0 \end{bmatrix}.$$

For hand calculations, the dependent controlled source terms are moved from the current vector to the admittance matrix. The following matrix equation shows the resulting matrix elements after the controlled source terms have been moved to the admittance matrix. After moving the terms, the node variables are ready to be solved from matrix equation using the Cramer's rule.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ \frac{\beta_{F1}}{r_{\pi 1}} & -\frac{\beta_{F1}}{r_{\pi 1}} & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & 0 & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Then from here

$$t_{11} = \frac{V_4}{V_s} = \frac{R_{C2} R_{E1} (\beta_{F1} + 1)}{(R_{C2} + R_F) [R_{E1} (\beta_{F1} + 1) + r_{\pi 1} + R_s] + R_{E1} (r_{\pi 1} + R_s)}. \quad (17)$$

As a single parameter this does not give that much information, but this is needed when composing the complete gain expression from individual feedback parameters.

3.4 t_{21} feedback parameter

Next we evaluate the feedback parameter t_{21} from its own small-signal model, where the output voltage source is set to zero ($X_o = V_o = 0$). This small-signal model is drawn in Figure 18, where the feedback from the output

voltage has been eliminated by setting the output voltage to zero. The elimination of the output voltage enables to analyse the situation where only the input source V_s is active and hence utilising the principle of superposition.

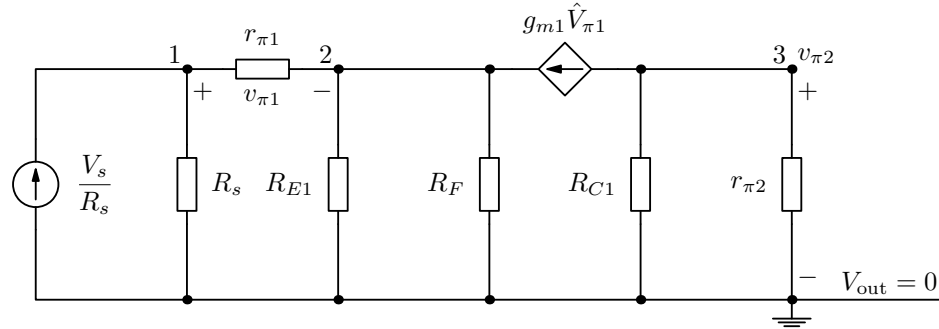


Figure 18: Small-signal model for evaluating the feedback parameter t_{21}

The dependent controlled source is chosen to be the first current-controlled current source and is identified in the figure and in the equations by the capitalised notation, $g_{m1}\hat{V}_{\pi1}$. The small-signal model of Figure 18 is mapped to the following matrix representation, where the admittance terms are collected to the main matrix and the current source terms are placed into the current vector. The voltage nodes in the voltage vector are the variables to be solved from this equation.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi1}} & -\frac{1}{r_{\pi1}} & 0 \\ -\frac{1}{r_{\pi1}} & \frac{1}{r_{\pi1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 \\ 0 & 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ g_{m1}\hat{V}_{\pi1} \\ -g_{m1}\hat{V}_{\pi1} \end{bmatrix} \quad (18)$$

For evaluation purposes it is convenient that all controlled source terms from the current vector are moved to the admittance matrix. Because the matrix equation is just a set of linear equations, the controlled source terms are subtracted or added to both sides of the equation. It is also necessary to identify to which voltage node variables the controlled source term relates to. Based on this discussion, the matrix equation (19) is readily modified in the desired form.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 \\ \frac{\beta_{F1}}{r_{\pi 1}} & -\frac{\beta_{F1}}{r_{\pi 1}} & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

Since $v_{\pi 1}$ is not directly any node variable, it has to be calculated as the difference $V_1 - V_2$. The feedback parameter t_{21} is then found to be

$$t_{21} = \frac{v_{\pi 1}}{V_s} = \frac{V_1 - V_2}{V_s} = \frac{r_{\pi 1}(R_{E1} + R_F)}{(r_{\pi 1} + R_s)(R_F + R_{E1}) + (\beta_{F1} + 1)R_{E1}R_F}. \quad (20)$$

3.5 Return ratio T and t_{12} feedback parameter

As already noted, as a single parameter the t_{21} does not tell much about the properties of the circuit. Because the aim is to find the expressions for β and T , the battle with the equations continues. For evaluating T and t_{12} the input source needs to be suppressed ($X_s = V_s$) and let the output source be active in turn. The small-signal model of Figure 19 provides a suitable model for the job.

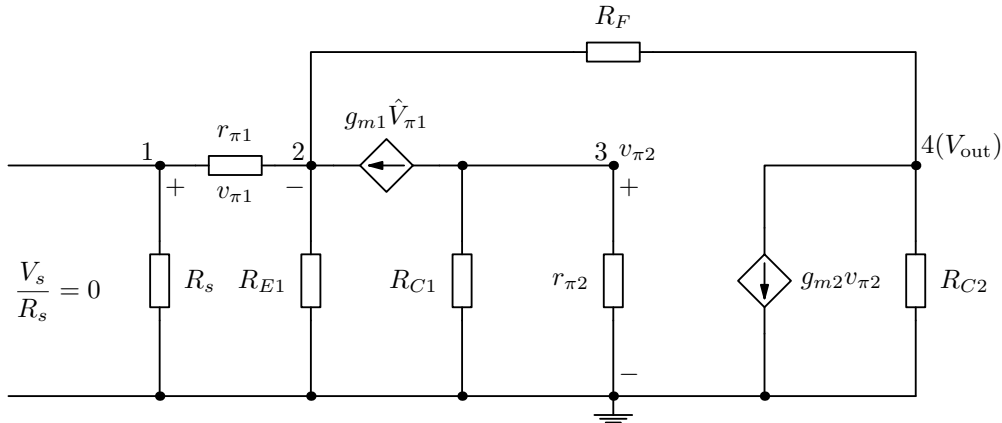


Figure 19: Small-signal model for feedback parameters T and t_{12}

From this model, the following matrix equation is written:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{1}{r_{\pi 1}} & \frac{1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ 0 & 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & 0 & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ g_{m1} \hat{V}_{\pi 1} \\ -g_{m1} \hat{V}_{\pi 1} \\ -g_{m2} \hat{V}_{\pi 2} \end{bmatrix}.$$

To ease out the evaluation process, all but one of the dependent controlled source terms are moved from the current vector to the admittance matrix. After moving the terms, the node variables are ready to be solved from matrix equation using the Cramer's rule.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ 0 & 0 & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & g_{m2} & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_{m1} \hat{V}_{\pi 1} \\ 0 \end{bmatrix}.$$

To find the return ratio T , the fraction $-\frac{v_{\pi 1}}{\hat{V}_{\pi 1}} = -\frac{V_1 - V_2}{\hat{V}_{\pi 1}}$ needs to be solved. The result of the lengthy calculation is

$$T = \frac{\beta_{F1} R_{C1} \beta_{F2} R_{C2}}{R_{C1} + r_{\pi 2}} \frac{R_{E1}}{(R_{C2} + R_{E1} + R_F)(R_s + r_{\pi 1}) + (\beta_{F1} + 1)R_{E1}(R_{C2} + R_F)}.$$

The same circuit model can be reused to evaluate the feedback parameter t_{12} , which is defined as $\frac{V_4}{\hat{V}_{\pi 1}}$. The equation for t_{12} is

$$t_{12} = \frac{\beta_{F1} R_{C1} \beta_{F2} R_{C2}}{R_{C1} + r_{\pi 2}} \frac{\frac{1}{r_{\pi 1}} [(r_{\pi 1} + R_s)(R_F + R_{E1}) + (\beta_{F1} + 1)R_{E1}R_F]}{(R_{C2} + R_{E1} + R_F)(R_s + r_{\pi 1}) + (\beta_{F1} + 1)R_{E1}(R_{C2} + R_F)}.$$

3.6 t_{22} , feedback factor β and gain factor K

The t_{22} parameter is obtained by dividing T by t_{12} . Hence,

$$t_{22} = -\frac{T}{t_{12}} = -\frac{r_{\pi 1} R_{E1}}{(r_{\pi 1} + R_s)(R_F + R_{E1}) + (\beta_{F1} + 1)R_{E1}R_F}.$$

Then finally the feedback factor β is obtained by dividing t_{22} by t_{21} and the result of this calculation is

$$\beta = \frac{t_{22}}{t_{21}} = -\frac{R_{E1}}{R_{E1} + R_F}.$$

From here it follows that the approximate gain factor

$$K = -\frac{1}{\beta} = \frac{R_{E1} + R_F}{R_{E1}}.$$

So here we got the expected result, which could have been guessed by directly looking at the circuit diagram. Here one did not even need to make any simplifications during the evaluation process, therefore the resistive feedback network does not interact with any of the surrounding passive components. One interesting fact is that the feedback factor is negative in this case. This actually a good thing because the output voltage in this circuit is connected as positive feedback to the emitter of the first transistor. From there the signal is inverted to the input side and from there comes the minus sign. In the current-shunt example the output signal was already inverted with respect to the input, that is why the feedback factor there was positive.

Now that all the feedback parameters have been calculated, the complete gain expression for the voltage gain A_v can be constructed as:

$$A_v = \frac{t_{11} + KT}{1 + T},$$

and with the calculated parameters this gives the exact gain of the complete circuit.

3.7 Input and output impedance

Finally the input impedance of the voltage-series feedback circuit is analysed just for the sake of completeness. Following the standard procedure, the dead-system impedance is evaluated from the model where the dependent controlled source is suppressed. This model is drawn into Figure 20. The dead-system model is identical to the model from where the t_{11} parameter was evaluated. Due to the local feedback loop around Q_1 , transistor Q_2 is taken off to break the amplification loop.

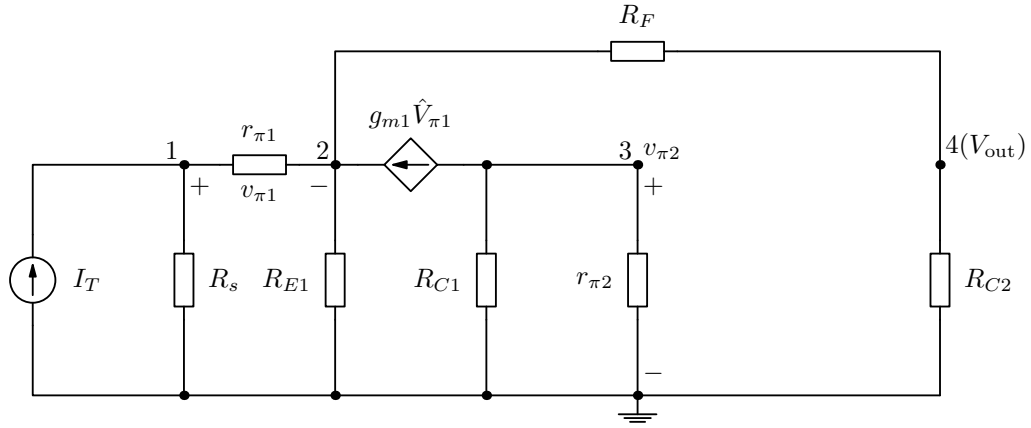


Figure 20: Small-signal model for evaluating the input impedance

Again the test current source is ideal, so the imagined source resistor R_s can be considered as infinite resistance and it can be taken out of the circuit. With this in mind, the matrix equation to evaluate the input impedance is very similar to the matrix for t_{11} . Here we directly write down the form, where the controlled current terms are already moved into the admittance matrix:

$$\begin{bmatrix} \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ \frac{\beta_{F1}}{r_{\pi 1}} & -\frac{\beta_{F1}}{r_{\pi 1}} & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & 0 & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_T \\ 0 \\ 0 \\ 0 \end{bmatrix} .$$

After solving this equation using Cramer's rule, the dead-system impedance reads

$$Z_{ID} = \frac{V_1}{I_T} = r_{\pi 1} + \frac{(R_{C2} + R_F)R_{E1}(\beta_{F1} + 1)}{R_{C2} + R_F + R_{E1}}$$

Now that we know the dead-system input impedance, next one needs to analyse the situations for $R_s \rightarrow \infty$ and $R_s \rightarrow 0$ in the expression of the return ratio. Setting $R_s = \infty$ results to $T = 0$ and $R_s = 0$ results to a reasonable value. Hence, the input impedance with feedback is defined as

$Z_{IF} = Z_{ID}(1 + T(R_s \rightarrow 0))$, where

$$T(R_s \rightarrow 0) = \frac{\beta_{F1} R_{C1} \beta_{F2} R_{C2}}{R_{C1} + r_{\pi 2}} \frac{R_{E1}}{(R_{C2} + R_{E1} + R_F) r_{\pi 1} + (\beta_{F1} + 1) R_{E1} (R_{C2} + R_F)}.$$

A direct conclusion is that voltage-series type of feedback increases the input impedance by the factor of $(1 + T)$.

The output impedance is evaluated in similar fashion. The small-circuit model for output impedance looks as drawn in Figure 21. The resistor R_{C2} is considered to be outside the point where the amplifiers output impedance is evaluated (looking into the amplifier). So we can utilise R_{C2} as the source resistor for our ideal test current source I_T .

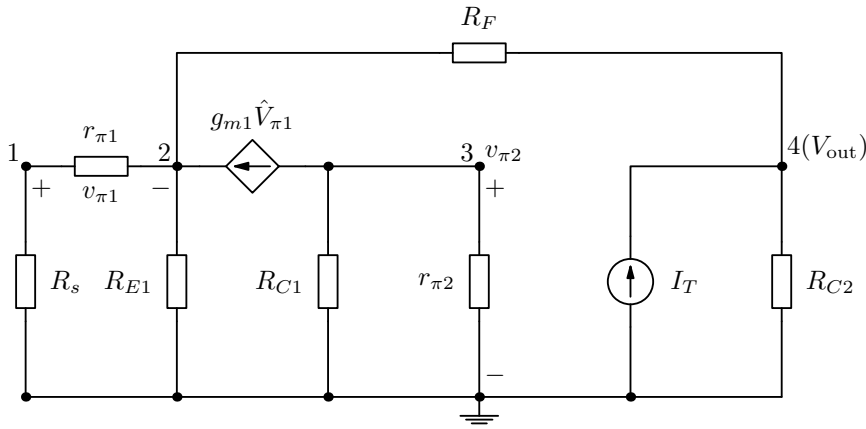


Figure 21: Small-signal model for evaluating the output impedance

Since the requirement is to have an ideal test source, we set $R_{C2} = \infty$ and just like R_s in the input impedance evaluation, R_{C2} can be taken out from the circuit under analysis. The matrix equation to evaluate the output impedance is

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & 0 & 0 \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_{E1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ \frac{\beta_{F1}}{r_{\pi 1}} & -\frac{\beta_{F1}}{r_{\pi 1}} & \frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} & 0 \\ 0 & -\frac{1}{R_F} & 0 & \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_T \end{bmatrix}.$$

After a boring process of manual calculations, the dead-system output impedance can be written as

$$Z_{OD} = R_F + \frac{R_{E1} \frac{r_{\pi 1} + R_s}{\beta_{F1} + 1}}{R_{E1} + \frac{r_{\pi 1} + R_s}{\beta_{F1} + 1}}.$$

Setting $R_{C2} = 0$ results to $T = 0$ and $R_{C2} = \infty$ gives the loop gain T a reasonable value. Hence, the output impedance with feedback is defined as

$$Z_{OF} = \frac{Z_{OD}}{1 + T(R_{C2} \rightarrow \infty)}, \quad (21)$$

and the output impedance looking after R_{C2} is the parallel impedance

$$Z'_{OF} = \frac{R_{C2} Z_{OF}}{R_{C2} + Z_{OF}}. \quad (22)$$

The reason for covering this amplifier configuration in such a complete fashion is that the derived equations can be used for calculating the gain of the circuit very accurately and the equations can be used to understand what is going on in different parts of the circuit. Furthermore, for example the White emitter follower is a direct application of this circuit and taking $R_{E1} = \infty$ (very large in numerical analysis) and $R_F = 0.0001$ (very small in numerical analysis) in the derived equations, they give accurate results for the White emitter follower as well. In that application the usefulness of the accurate input and output impedance equations are seen in practise.

From another point of view, Figure 22 indicates a situation where the Miller theorem is used to break the feedback loop into two separate resistors. The rescaled feedback resistor R_F appears in parallel with the load resistor (look at R_{C2} in the small-signal model in this case) and also in parallel with the emitter resistor R_{E1} . The positive feedback reflects R_F as *negative impedance* in parallel with R_{E1} . This negative impedance increases the total emitter resistance. The final step towards negative feedback to the input terminals is arranged as a *local* feedback loop from the emitter of Q_1 .

4 GLOBAL FEEDBACK VERSUS LOCAL FEEDBACK

The concepts global and local feedback basically relate to the amount of components which the feedback loop encloses within. If the feedback loop is taken from the output to the input over one single transistor, the feedback is localised and therefore the feedback is commonly referred to as local feed-

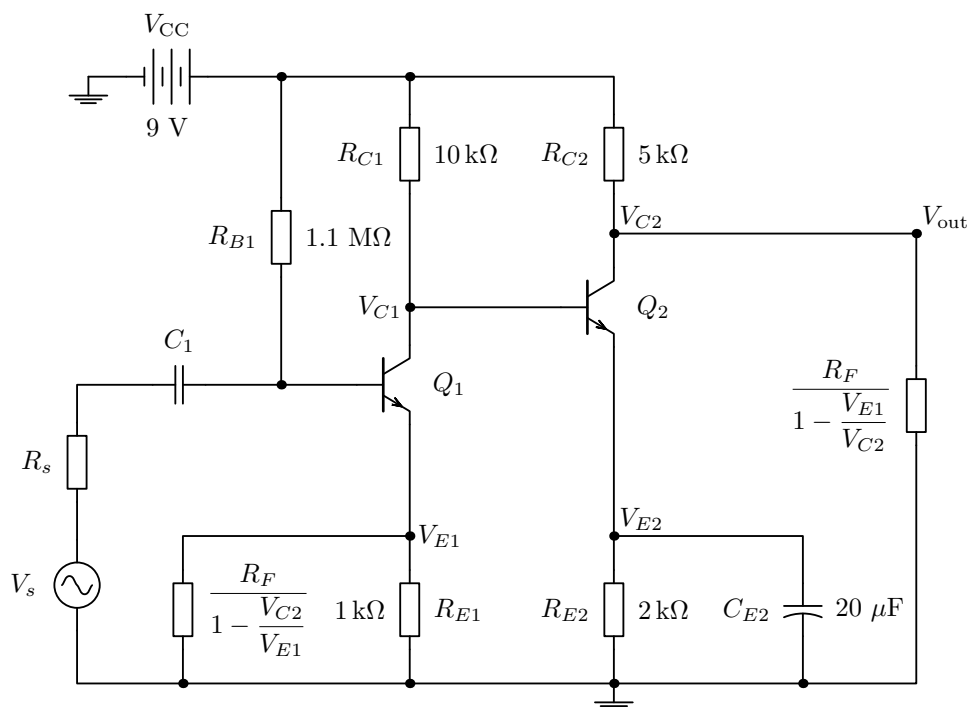


Figure 22: The feedback impedance split according to the Miller theorem

back. If the feedback loop extends over a complete circuit stage with several amplifier elements, then the feedback is categorised as global feedback.

Both of the examples already given in this context are more or less examining the feedback in the global scope. The most common type of local feedback is found from a simple single-stage BJT amplifier with an emitter resistor (see Figure 23). Typically the emitter resistor is bypassed for AC-signals with a capacitor, but when the capacitor is not there, the effective input signal is facing a feedback loop from the emitter to the input.

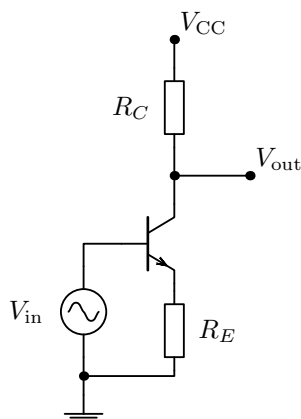


Figure 23: A BJT amplifier with an unbypassed emitter resistor

Figure 24 indicates that the local feedback structure of an unbypassed CE-stage is of the series-series type, where the current from the output is sampled and converted into voltage in the feedback loop. The voltage due to the output current over the emitter resistor is therefore seen as inverted voltage at the input side of the transistor.

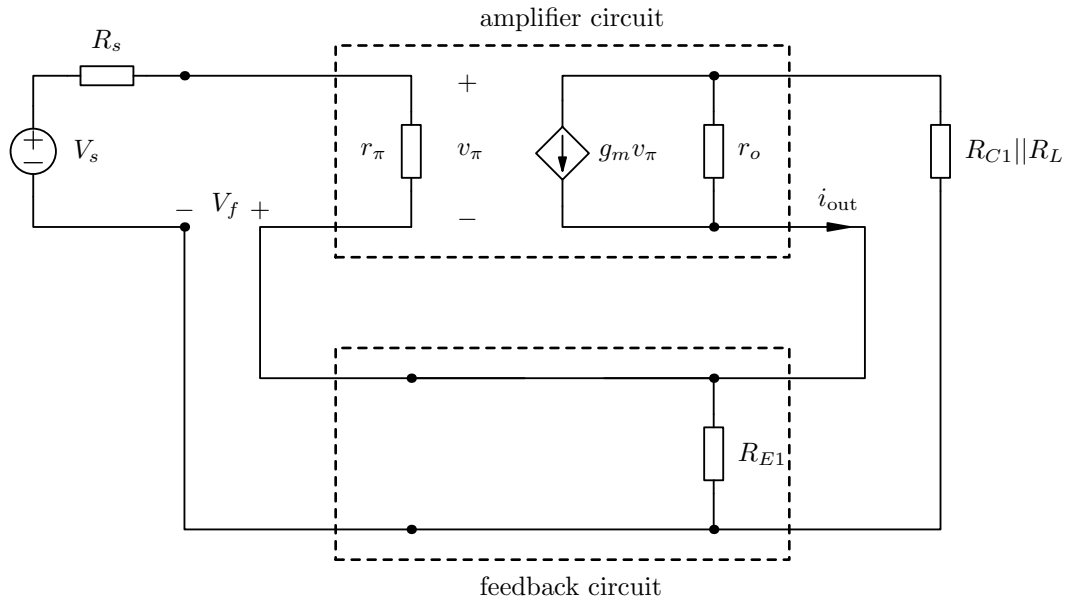


Figure 24: A series-series feedback network of the BJT with an emitter resistor

This series-series CE stage amplifier is analysed more closely in section 7.

5 BONUS: SHUNT-SHUNT FEEDBACK CIRCUIT

This is a quick addition after noting the benefits of the collector-to-base feedback configuration of a single BJT amplifier. This configuration is drawn in Figure 25. The resistor between the collector and the base serves both as a DC bias resistor and an AC signal feedback path.

5.1 DC analysis

First let's check the DC analysis of this circuit. Figure 26 shows the components, voltage nodes and currents which participate to the biasing scheme. It is easy to see that there is only one unknown voltage node V_C , so it should be relatively easy to derive the bias equations for this circuit. The current equation for node V_C can be written as

$$I'_C = I_C + I' = \beta_F(I' - I) + I' = (\beta_F + 1)I' - \beta_F I, \quad (23)$$

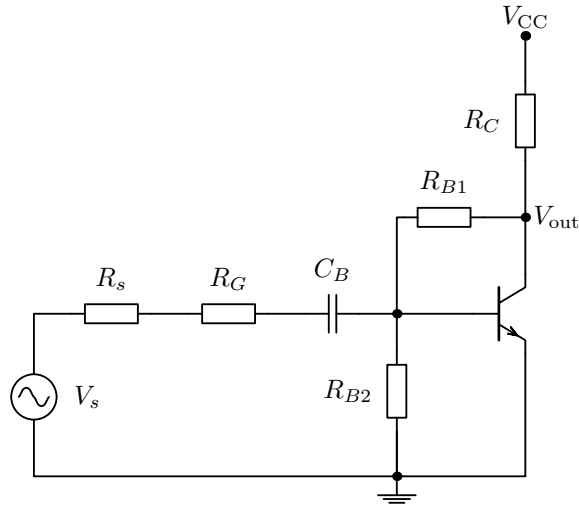


Figure 25: A BJT amplifier with collector-to-base bias and feedback

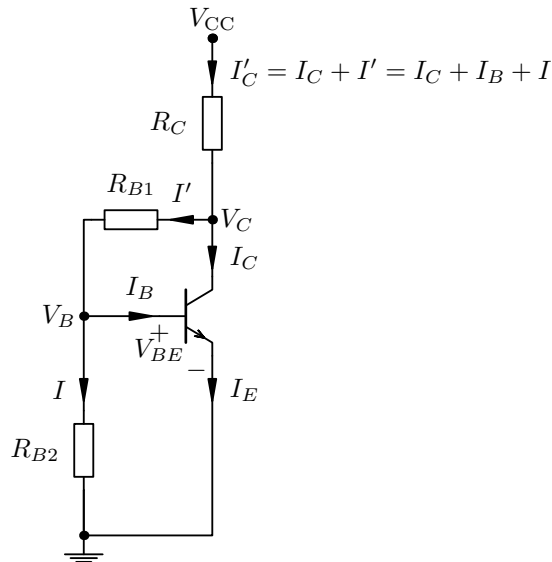


Figure 26: DC model for biasing calculations

and the currents can be expressed using voltages and resistances as follows:

$$I'_C = \frac{V_{CC} - V_C}{R_C} \quad ; \quad I' = \frac{V_C - V_{BE}}{R_{B1}} \quad ; \quad I = \frac{V_{BE}}{R_{B2}}. \quad (24)$$

From here one can deduce that the collector bias voltage

$$V_C = \frac{\frac{V_{CC}}{R_C} + \frac{(\beta_F + 1)V_{BE}}{R_{B1}} + \frac{\beta_F V_{BE}}{R_{B2}}}{\frac{1}{R_C} + \frac{(\beta_F + 1)}{R_{B1}}}. \quad (25)$$

Other values can be calculated based on the need. This concludes the DC bias analysis for this circuit.

5.2 AC analysis with feedback parameters

The small-signal model corresponding to the circuit of Figure 25 is shown in Figure 27. Let's analyse the basic feedback parameters, the feedback factor β and the return ratio T , for this small-signal model.

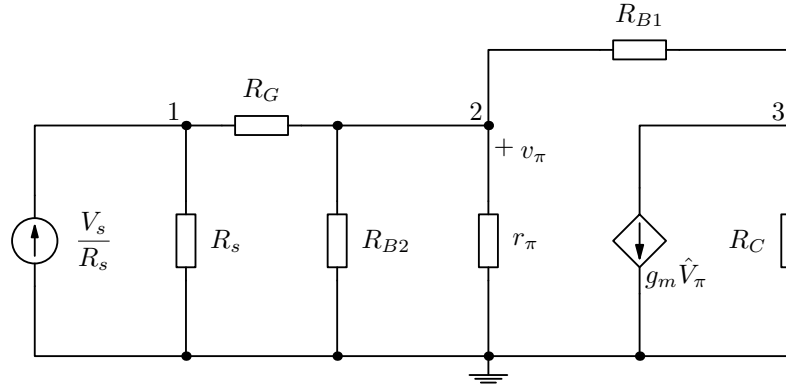


Figure 27: A small-signal model of the collector-to-base feedback BJT amplifier

According to the standard procedure, first we calculate the feedback parameter t_{21} , which requires that the output voltage node is set to zero (ground potential) before evaluation. This leads to the following matrix equation:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_G} & -\frac{1}{R_G} \\ -\frac{1}{R_G} & \frac{1}{R_G} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{r_\pi} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ 0 \end{bmatrix}.$$

Solving the matrix equation for the node voltage V_2 , this gives:

$$t_{21} = \frac{V_2}{V_s} = \frac{R_{B1}R_{B2}r_\pi}{(r_\pi + R_{B2})(R_{B1}R_G + R_{B1}R_s) + r_\pi R_{B2}(R_s + R_{B1} + R_G)}.$$

Next we evaluate the return ratio T , which in turn requires the input source to be set to zero. When the input current source is set to zero, it leaves an open circuit and from that configuration the following matrix equation is obtained:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_G} & -\frac{1}{R_G} & 0 \\ -\frac{1}{R_G} & \frac{1}{R_G} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{r_\pi} & -\frac{1}{R_{B1}} \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_m \hat{V}_\pi \end{bmatrix}.$$

After solving for V_2 , the equation for the return ratio T is found to be

$$T = -\frac{V_2}{\hat{V}_\pi} = \frac{g_m r_\pi R_{B2} R_C (R_s + R_G)}{(r_\pi + R_{B2})(R_{B1} + R_C)(R_G + R_s) + r_\pi R_{B2}(R_s + R_G + R_C + R_{B1})}.$$

One can use the same matrix equation to solve for node voltage V_3 and get the feedback parameter t_{12} , which comes out as

$$t_{12} = \frac{V_3}{\hat{V}_\pi} = -\frac{g_m R_C [(r_\pi + R_{B2})(R_{B1} R_G + R_{B1} R_s) + r_\pi R_{B2}(R_s + R_{B1} + R_G)]}{(r_\pi + R_{B2})(R_{B1} + R_C)(R_G + R_s) + r_\pi R_{B2}(R_s + R_G + R_C + R_{B1})}.$$

For completeness one can also evaluate the dead-system gain parameter t_{11} . It is evaluated as a normal transfer function $\frac{V_o}{V_s}$, but the controlled source is suppressed (removed) from the circuit. In this case, the same matrix equation which was used for evaluating T and t_{12} can be used for calculating t_{11} as well. The result is

$$t_{11} = \frac{r_\pi R_{B2} R_C}{(r_\pi + R_{B2})(R_{B1} + R_C)(R_G + R_s) + r_\pi R_{B2}(R_s + R_G + R_C + R_{B1})}.$$

The obtained t-parameters can now be used for evaluating the open-loop gain, which is $A_{OL} = t_{11} + t_{12}t_{21}$. However, in this context we don't really care about the open-loop gain. More importantly the reverse transmission transfer function (or the feedback factor) β can now be evaluated using the t-parameters, but we still need the parameter t_{22} for that. One gets the missing parameter from

$$t_{22} = -\frac{T}{t_{12}} = \frac{r_\pi R_{B2}(R_s + R_G)}{(r_\pi + R_{B2})(R_{B1} R_G + R_{B1} R_s) + r_\pi R_{B2}(R_s + R_{B1} + R_G)},$$

and then

$$\beta = \frac{t_{22}}{t_{21}} = \frac{R_s + R_G}{R_{B1}} \quad \Rightarrow \quad K = -\frac{R_{B1}}{R_s + R_G}.$$

The gain factor K is the negative inverse of the feedback factor by definition, meaning that

$$K = -\frac{1}{\beta}.$$

It is important to note that since the feedback evaluation was completely done using voltage signals $\frac{V_x}{V_y}$, this gain parameter is valid only for describing voltage gain. According to the general theory of feedback, the total gain obtained from the circuit with feedback is defined by the return ratio T and the voltage gain factor K as

$$A_v = \frac{t_{11} + KT}{1 + T}. \quad (26)$$

If evaluated numerically, this result will give exactly the same values as if one would have directly calculated the transfer function for this circuit.

5.3 Input and output impedance

The input and output impedances are evaluated using the Blackman's impedance formula. Regarding the input impedance, the Blackman's formula leads to the conclusion that the input impedance with feedback is practically the same as the dead-system input impedance, which is dominated by R_1 (since R_s is not considered to be part of the feedback impedance). The way to derive the equation for input impedance is to evaluate the loop-gain T when setting $R_s = 0$ and $R_s = \infty$ (short-circuit and open-circuit T for the Blackman's formula). In this case neither of these terms are zero. For setting $R_s = \infty$, one gets

$$T(R_s \rightarrow \infty) = \frac{g_m r_\pi R_{B2} R_C}{(r_\pi + R_{B2})(R_{B1} + R_C) + r_\pi R_{B2}},$$

and for $R_s = 0$ we get

$$T(R_s \rightarrow 0) = \frac{g_m r_\pi R_{B2} R_C R_G}{R_G(r_\pi + R_{B2})(R_{B1} + R_C) + r_\pi R_{B2}(R_G + R_C + R_{B1})}.$$

The dead-system (controlled current source suppressed) input impedance is found to be

$$Z_{DI} = R_G + \frac{r_\pi R_{B2}(R_{B1} + R_C)}{r_\pi R_{B2} + (r_\pi + R_{B2})(R_{B1} + R_C)}$$

and then according to Blackman's formula, the input impedance with feedback is

$$Z_{IF} = Z_{DI} \frac{1 + T(R_s \rightarrow 0)}{1 + T(R_s \rightarrow \infty)}.$$

The output impedance is obtained in a similar fashion. Setting $R_C = 0$ in the expression of T (short circuit) gives nothing reasonable. Hence, the only meaningful value must come from setting $R_C = \infty$, and indeed

$$T(R_C \rightarrow \infty) = \frac{g_m r_\pi R_{B2}(R_s + R_G)}{(r_\pi + R_{B2})(R_s + R_G) + r_\pi R_{B2}}.$$

The dead-system output impedance is found to be

$$Z_{DO} = R_{B1} + \frac{r_\pi R_{B2}(R_s + R_G)}{r_\pi R_{B2} + (r_\pi + R_{B2})(R_s + R_G)}$$

and using the Blackman's impedance formula, the output impedance with feedback is obtained from

$$Z_{OF} = \frac{Z_{DO}}{1 + T(R_C \rightarrow \infty)}.$$

Actually, this is only the output impedance when looking into the transistor before the collector resistor R_C . This means that the collector resistor is

considered as a load for this circuit. The output impedance when looking after R_C is therefore

$$Z'_{OF} = \frac{R_C Z_{OF}}{R_C + Z_{OF}},$$

that is, the parallel impedance of R_C and Z_{OF} .

When using the shunt-shunt feedback topology, the amplifier becomes a transresistance amplifier, which expects current to the input side and amplifies this current into a voltage output signal.

5.4 Circuit with R_E included

The obtained results for the circuit without R_E can be extended to cover the circuit with R_E using a neat little trick. Beware that this trick does not apply for every circuit, but quite often it works nicely. When one has evaluated T , K and t_{11} for the circuit without R_E , the circuit with R_E included can be modelled by making the following changes:

1. If the expression for T has the combination $g_m r_\pi$ in the numerator, assign that as β_F .
2. Then assign $r_\pi = r_\pi + (\beta_F + 1)R_E$ for all the remaining occurrences of r_π in equations of t_{11} , T and K . After this one gets the accurate voltage gain results from equation (26).

Note that this same approach works for the impedance evaluation as well. Just make sure that the combinations of $g_m r_\pi$ in the numerators of T are identified and marked as β_F before making the adjustment to r_π in all equations.

6 EXTRA BONUS: EMITTER FOLLOWER FEEDBACK CIRCUIT

This is another quick addition to cover the feedback analysis of the common-collector configuration with base resistors included. This circuit is a good example of a shunt-series feedback type, which was covered already earlier. The emitter follower circuit for analysis is shown in Figure 28.

We will skip the DC analysis for this circuit and only note that the DC analysis is exactly the same as for the four-resistor common-emitter amplifier. For

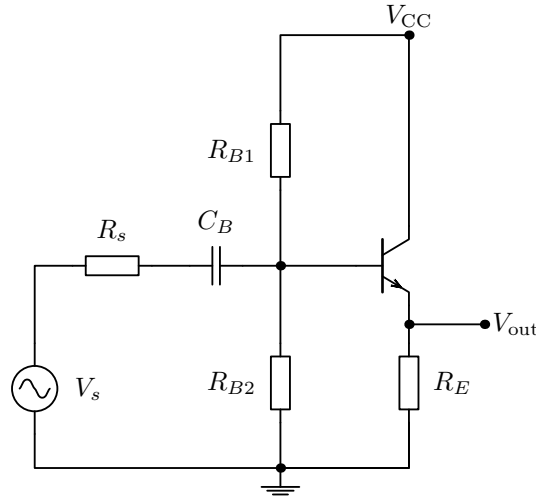


Figure 28: A common-collector BJT amplifier

reference we just recall that the base bias current is calculated from

$$I_B = \frac{V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}} - V_{BE}}{\frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}} + (\beta_F + 1) R_E},$$

and all other voltages and currents can be processed from there.

6.1 AC analysis with feedback parameters

The small-signal model corresponding to the circuit of Figure 28 is shown in Figure 29. The voltage node V_2 is the output node V_{out} of Figure 28. The base resistors have been combined to a single resistor R_B , which equals the parallel resistance of R_{B1} and R_{B2} , that is,

$$R_B = \frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}}.$$

Let's analyse the basic feedback parameters, the feedback factor β and the return ratio T using this small-signal model.

According to the standard procedure, first we calculate the feedback parameter t_{21} , which requires that the output voltage node V_2 is set to zero (ground potential) before evaluation. This leads to the following simple equation:

$$V_1 = \frac{V_s}{R_s} \left(\frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} \right).$$

Solving the voltage relation between V_1 and V_s yields

$$t_{21} = \frac{V_1}{V_s} = \frac{R_B r_\pi}{R_B(R_s + r_\pi) + R_s r_\pi}.$$

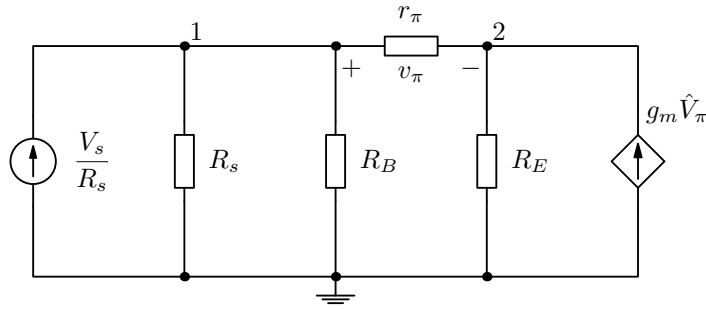


Figure 29: A small-signal model of the common-collector BJT amplifier

Next we evaluate the return ratio T , which in turn requires the input source to be set to zero. When the input current source in the small-signal model is set to zero, it leaves an open circuit and from that configuration the following matrix equation is obtained:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} \\ -\frac{1}{r_\pi} & \frac{1}{r_\pi} + \frac{1}{R_E} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ g_m \hat{V}_\pi \end{bmatrix}.$$

Since v_π is now $V_1 - V_2$, one needs to solve both nodes from the matrix equation. Luckily the matrix has only two rows and two columns, so there is not too much work to be done. After solving for V_2 , the equation for the return ratio T is found to be

$$T = -\frac{V_1 - V_2}{\hat{V}_\pi} = \frac{g_m r_\pi R_E (R_s + R_B)}{R_B (R_s + r_\pi + R_E) + R_s (r_\pi + R_E)}.$$

One can use the same matrix equation to solve for node voltage V_2 and get the feedback parameter t_{12} , which comes out as

$$t_{12} = \frac{V_2}{\hat{V}_\pi} = \frac{g_m R_E [R_B (R_s + r_\pi) + R_s r_\pi]}{R_B (R_s + r_\pi + R_E) + R_s (r_\pi + R_E)}.$$

For completeness one can also evaluate the dead-system gain parameter t_{11} . It is evaluated as a normal transfer function $\frac{V_o}{V_s}$, but the controlled source is suppressed (removed) from the circuit. In this case, the same matrix equation which was used for evaluating T and t_{12} can be used for calculating t_{11} as well. The result is

$$t_{11} = \frac{R_B R_E}{R_B (R_s + r_\pi + R_E) + R_s (r_\pi + R_E)}.$$

The obtained t-parameters can now be used for evaluating the open-loop gain, which is $A_{OL} = t_{11} + t_{12}t_{21}$. However, in this context we don't really

care about the open-loop gain. More importantly the reverse transmission transfer function (or the feedback factor) β can now be evaluated using the t -parameters, but we still need the parameter t_{22} for that. One gets the missing parameter from

$$t_{22} = -\frac{T}{t_{12}} = -\frac{r_{\pi}(R_s + R_B)}{R_B(R_s + r_{\pi}) + R_s r_{\pi}},$$

and then

$$\beta = \frac{t_{22}}{t_{21}} = -\frac{R_s + R_B}{R_B} \Rightarrow K = \frac{R_B}{R_s + R_B}.$$

It is important to note that since the feedback evaluation was completely done using voltage signals $\frac{V_x}{V_y}$, this gain parameter is valid only for describing voltage gain. According to the general theory of feedback, the total gain obtained from the circuit with feedback is defined by the return ratio T and the voltage gain factor K as

$$A_F = A_v = \frac{t_{11} + KT}{1 + T}.$$

If evaluated numerically, this result will give exactly the same values as if one would have directly calculated the transfer function for this circuit.

6.2 Input and output impedance

The input and output impedances are evaluated using the Blackman's impedance formula. The way to derive the equation for input impedance is to calculate the dead-system input impedance and evaluate the return ratio T when setting $R_s = 0$ and $R_s = \infty$ (short-circuit and open-circuit T for the Blackman's formula). When $R_s = \infty$, the return ratio becomes

$$T(R_s \rightarrow \infty) = \frac{g_m r_{\pi} R_E}{R_B + r_{\pi} + R_E}.$$

When settings $R_s = 0$ in the original equation for T , this gives

$$T(R_s \rightarrow 0) = \frac{g_m r_{\pi} R_E}{r_{\pi} + R_E}.$$

The dead-system (controlled current source suppressed) input impedance is found to be

$$Z_{DI} = \frac{R_B(r_{\pi} + R_E)}{R_B + r_{\pi} + R_E}$$

and then according to Blackman's formula, the input impedance with feedback is

$$Z_{IF} = \frac{Z_{DI}(1 + T(R_s \rightarrow 0))}{1 + T(R_s \rightarrow \infty)}.$$

The output impedance is obtained in a similar fashion. Setting $R_E = 0$ in the expression of T (short circuit) gives nothing reasonable. Hence, the only meaningful value must come from setting $R_E = \infty$, and indeed

$$T(R_E \rightarrow \infty) = g_m r_\pi.$$

The dead-system output impedance is found to be

$$Z_{DO} = r_\pi + \frac{R_s R_B}{R_s + R_B}$$

and using the Blackman's impedance formula, the output impedance with feedback is obtained from

$$Z_{OF} = \frac{Z_{DO}}{1 + T(R_E \rightarrow \infty)}.$$

Actually, this is only the output impedance when looking into the transistor before the emitter resistor R_E . This means that the emitter resistor is considered as a load for this circuit. The output impedance when looking after R_E is therefore

$$Z'_{OF} = \frac{R_E Z_{OF}}{R_E + Z_{OF}},$$

that is, the parallel impedance of R_E and Z_{OF} .

7 SUPER BONUS: COMMON EMITTER FEEDBACK CIRCUIT

This section needs to be added to reveal the caveats in the systematic approach of using matrices and determinants for circuit analysis. In some cases one ends up having an ambiguous determinant, where one row or column is all zeros, but it can be worked around not to have all zeros by adding rows or columns and get a valid answer. This is nasty, but can be worked out by using our brains for even once.

The common-emitter BJT amplifier without a bypass capacitor beside the emitter resistor implements a series-series feedback configuration, where the emitter resistor works as the feedback actuator just like in the case of the emitter follower. A series-series feedback makes the circuit behave like a transconductance amplifier, where a voltage signal at the input is converted as a current signal at the output. However, we will analyse the feedback parameters using current sources both in input and output.

7.1 AC analysis with feedback parameters

Let's skip the DC analysis and get straight to the point to evaluate the feedback parameters. The small-signal model of the common-emitter amplifier is

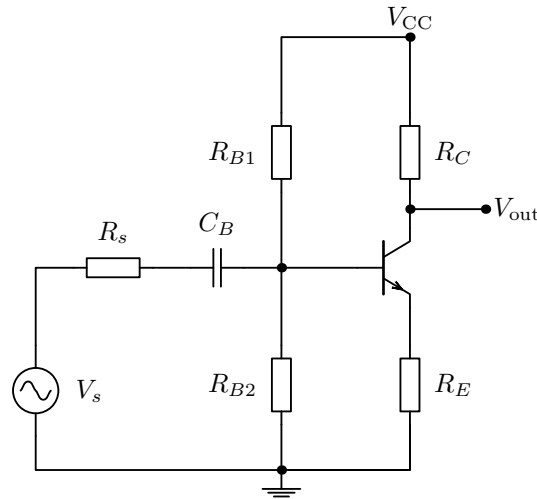


Figure 30: A common-emitter BJT amplifier

shown in Figure 31. The voltage node V_3 is the output node V_{out} of Figure 30. The base resistors have been combined to a single resistor R_B , which equals the parallel resistance of R_{B1} and R_{B2} , that is,

$$R_B = \frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}}.$$

Let's analyse the basic feedback parameters, the feedback factor β and the return ratio T using this small-signal model.

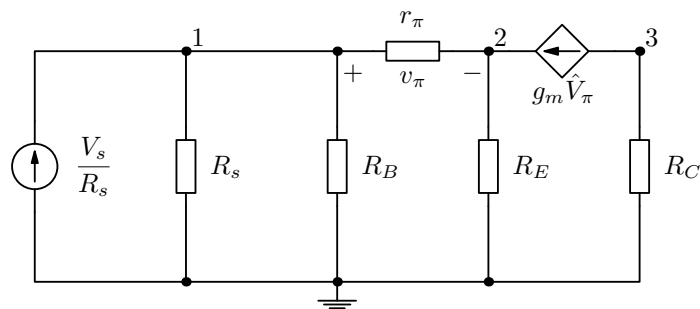


Figure 31: A small-signal model of the common-emitter BJT amplifier

According to the standard procedure, first we calculate the feedback parameter t_{21} , which in this case requires that the output current is set to zero by cutting out the controlled source before evaluation. The circuit for evaluating t_{21} is shown in Figure 32. Note that even though the situation at the output is similar to the current-shunt example, here it is enough to remove the controlled source, because already this will prevent the amplified signal to be fed back to the input side.

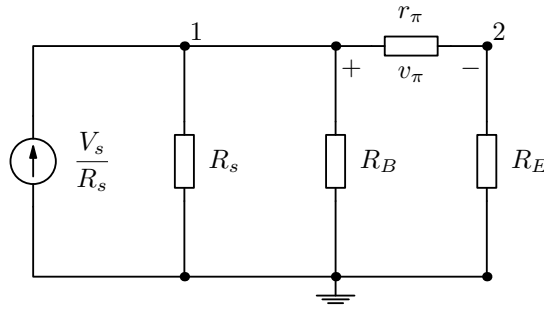


Figure 32: Circuit for evaluating the t_{21} feedback parameter

The evaluation of the feedback parameter t_{21} leads to the following equation using the current divider relation:

$$i_{\pi} = \frac{V_s}{R_s} \frac{\frac{R_s R_B}{R_s + R_B}}{\frac{R_s R_B}{R_s + R_B} + r_{\pi} + R_E}.$$

This indicates the amount of current going through the transistor input resistance r_{π} . Solving the current relation between i_{π} and I_s , the feedback factor

$$t_{21} = \frac{i_{\pi}}{I_s} = \frac{R_s R_B}{(R_s + R_B)(r_{\pi} + R_E) + R_s R_B}.$$

Next we evaluate the return ratio T and the feedback parameter t_{12} , which in turn requires the input source to be set to zero. This circuit modification is indicated in Figure 33, where the input current source is removed from the circuit, and the rest of the small-signal model remains untouched.

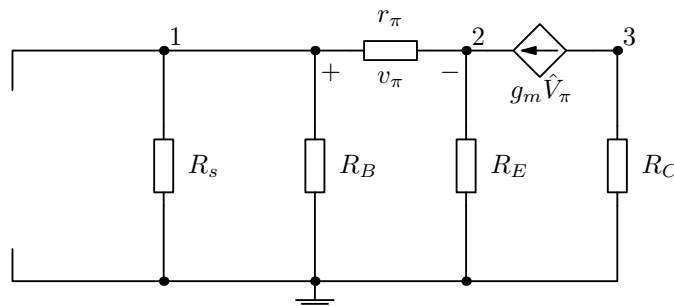


Figure 33: Circuit for evaluating the T and t_{12} feedback parameters

When the input current source in the small-signal model is set to zero, it leaves an open circuit and from that configuration the following matrix equa-

tion is obtained:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{1}{r_\pi} & \frac{1}{r_\pi} + \frac{1}{R_E} & 0 \\ 0 & 0 & \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \beta_F \hat{I}_\pi \\ -\beta_F \hat{I}_\pi \end{bmatrix}.$$

So yeah, now comes the sad part of the feedback story. One could try to solve the matrix equation by moving either one of the current terms into the admittance matrix and use Cramer's rule as always. Most likely the wrong answer would be obtained. To get both T and t_{12} correctly out from the same matrix equation, we do this trick:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{1}{r_\pi} & \frac{1}{r_\pi} + \frac{1}{R_E} & \frac{1}{R_C} \\ 0 & 0 & \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\beta_F \hat{I}_\pi \end{bmatrix}.$$

We added row 3 into row 2 to get rid of the other current term and being also farsighted we avoided an all zero column in Cramer's determinants. This is nasty, because it is far from being a systematic way of solving the loop gain. To get the matrix into the correct form, one needs to use traditional circuit analysis methods to verify the consistency of the results.

Anyways, since v_π is now $V_1 - V_2$, for the loop gain T one needs to solve both nodes 1 and 2 from the matrix equation. After some equation twisting, the return ratio T is found to be

$$T = -\frac{V_1 - V_2}{\hat{I}_\pi r_\pi} = -\frac{i_\pi}{\hat{I}_\pi} = \frac{\beta_F R_E (R_s + R_B)}{(R_B + R_s)(r_\pi + R_E) + R_s R_B}.$$

The result is the same as in the case of the emitter follower circuit although written here in a different way. One can use the same matrix equation to solve for node voltage V_3 and get the feedback parameter t_{12} , which comes out as

$$t_{12} = \frac{V_3}{\hat{I}_\pi R_C} = \frac{I_o}{\hat{I}_\pi} = -\beta_F.$$

Whoa, that is simple! One can verify the result easily by direct inspection on the small-signal model.

After solving T and t_{12} , those can be used for evaluating the remaining t_{22} parameter. One gets the missing parameter from

$$t_{22} = -\frac{T}{t_{12}} = -\frac{R_E(R_s + R_B)}{(R_B + R_s)(r_\pi + R_E) + R_s R_B}.$$

and then

$$\beta = \frac{t_{22}}{t_{21}} = \frac{I_s}{I_o} = -\frac{R_E}{\frac{R_s R_B}{R_s + R_B}}$$

$$K = -\frac{1}{\beta} = -\frac{I_o}{I_s} = \frac{R_s || R_B}{R_E}. \quad (27)$$

It is important to note that since the feedback evaluation was completely done using current signals $\frac{I_x}{I_y}$, this gain parameter is valid only for describing current gain. But, since $V_s = I_s R_s$ and $V_o = -I_o R_C$, we can turn the gain parameter to describe voltage gain. So,

$$K_{\text{voltage}} = \frac{I_o R_C}{I_s R_s} = \frac{V_o}{V_s} = -\frac{R_C}{R_E} \frac{R_B}{R_s + R_B} \approx -\frac{R_C}{R_E}. \quad (28)$$

The last approximation is valid when the source resistance R_s is much smaller than the base resistance R_B . If comparing to the results obtained from the emitter follower, the gain term is simply scaled by the factor $-\frac{R_C}{R_E}$.

for the sake of completeness, the feedback parameter t_{11} is evaluated as $\frac{I_o}{I_s}$ from the small-signal model with the controlled source removed. It seems that in this case $t_{11} = 0$, because the output current is zero after removing the controlled current source.

According to the general theory of feedback, the total gain obtained from the circuit with feedback is defined by the return ratio T and the voltage gain factor K as

$$A_F = A_i = \frac{t_{11} + KT}{1 + T}.$$

If all the parameters have been calculated using the current source representation, then this gain equation will spit out the current gain of the circuit. In this case it is fairly easy to convert the current relations into voltage relations and then the gain will be obtained for voltage signals. The K is presented in voltage form above and the loop gain T in voltage form is identical to the emitter follower loop gain, namely:

$$T = -\frac{V_1 - V_2}{\hat{V}_\pi} = \frac{g_m r_\pi R_E (R_s + R_B)}{(R_s + R_B)(r_\pi + R_E) + R_s R_B}.$$

Then,

$$A_F = A_v = \frac{KT}{1+T} = \frac{-\beta_F R_C R_B}{(R_s + R_B)[r_\pi + R_E(\beta_F + 1)] + R_s R_B}.$$

7.2 Input and output impedance

Also the input impedance is evaluated in the same manner as for the emitter follower. The output impedance needs the same reasoning as presented in the section covering the shunt-series direct-coupled amplifier. Since the transistor internal resistance r_o is assumed to be infinitely large, the output impedance is simply R_C .

8 ULTRA BONUS: COMMON EMITTER FEEDBACK WITH r_o

This section covers a practically useless BJT configuration, where an external resistor R_O is connected between the collector and emitter. It does not affect the DC bias in any way, because of the added (unlabelled) capacitor. In the AC model, R_O connects parallel with the BJT internal output resistance r_o . Figure 34 gives more details of the circuit under analysis.

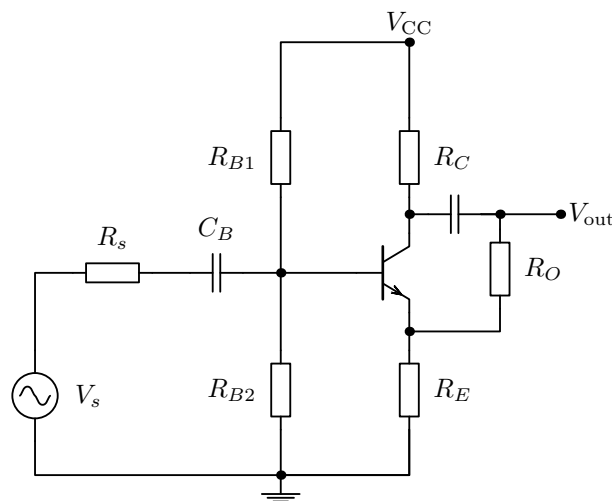


Figure 34: A common-emitter BJT amplifier with a resistor between collector and emitter

The reason for studying this circuit is the desire to find out if there is any feedback through the added R_O back to the input via the emitter resistor.

Basically this is still a series-series feedback configuration, but the uncertainty comes from the fact that R_O seems to be in a shunt connection in the

output side while R_E does the current sampling in a series configuration. We will try to analyse the feedback parameters using current sources both in input and output, the same way as was done with the basic common-emitter amplifier.

8.1 AC analysis with feedback parameters

The small-signal model of the common-emitter amplifier with added R_O is shown in Figure 31.

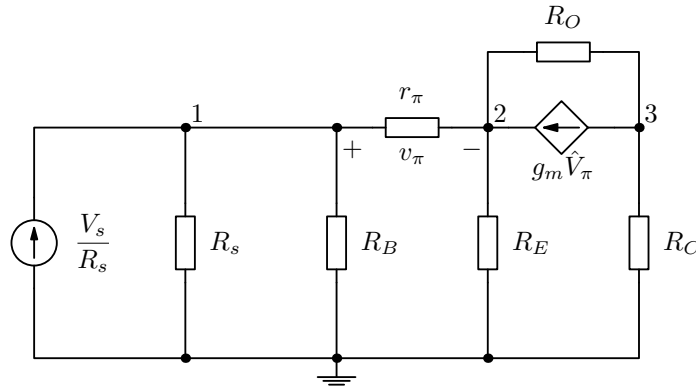


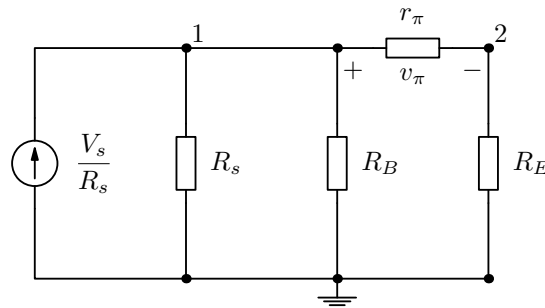
Figure 35: A small-signal model of the common-collector BJT amplifier with R_O

The voltage node V_3 is the output node V_{out} of Figure 35. The base resistors have been combined to a single resistor R_B , which equals the parallel resistance of R_{B1} and R_{B2} , that is,

$$R_B = \frac{R_{B1}R_{B2}}{R_{B1} + R_{B2}}.$$

First we calculate the feedback parameter t_{21} . The circuit for evaluating t_{21} is shown in Figure 36. The output current has been zeroed by cutting the connection to the controlled source and to R_O , which would carry some current even when the controlled source is removed. Another way of looking at it is that this way both voltage and current in the output is zeroed. Hence the small-signal model for t_{21} looks the same as in the case of the standard common-emitter amp.

But why aren't R_O and R_C included as a load after R_E ? The reason is that the loading of the feedback section is taken into account only when it connects back to the input side of the amplifier. In this case both R_O and R_C are in the

Figure 36: Circuit for evaluating the t_{21} feedback parameter

output side of the amplifier. This is the first hint that R_O is not a feedback resistor at all.

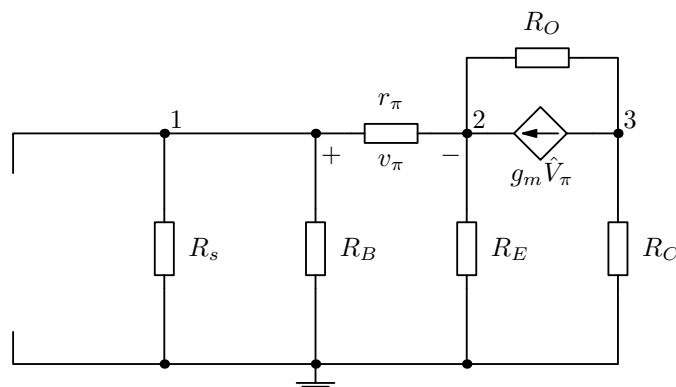
The evaluation of the feedback parameter t_{21} leads to the following equation using the current divider relation:

$$i_\pi = \frac{V_s}{R_s} \frac{\frac{R_s R_B}{R_s + R_B}}{\frac{R_s R_B}{R_s + R_B} + r_\pi + R_E}.$$

This indicates the amount of current going through the transistor input resistance r_π . Solving the current relation between i_π and I_s , the feedback factor

$$t_{21} = \frac{i_\pi}{I_s} = \frac{R_s R_B}{(R_s + R_B)(r_\pi + R_E) + R_s R_B}.$$

Next we evaluate the return ratio T and the feedback parameter t_{12} , which in turn requires the input source to be set to zero. This circuit modification is indicated in Figure 37, where the input current source is removed from the circuit, and the rest of the small-signal model remains untouched.

Figure 37: Circuit for evaluating the T and t_{12} feedback parameters

When the input current source in the small-signal model is set to zero, it leaves an open circuit and from that configuration the following matrix equation is obtained:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{1}{r_\pi} & \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_O} & -\frac{1}{R_O} \\ 0 & -\frac{1}{R_O} & \frac{1}{R_C} + \frac{1}{R_O} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \beta_F \hat{I}_\pi \\ -\beta_F \hat{I}_\pi \end{bmatrix}.$$

Using the same solving technique as in the case of the basic common-emitter amplifier, the return ratio is

$$T = -\frac{i_\pi}{\hat{I}_\pi} = \frac{\beta_F R_O R_E (R_s + R_B)}{(R_C + R_O)[(R_s + R_B)(r_\pi + R_E) + R_s R_B] + R_E[(R_s + R_B)r_\pi + R_s R_B]}.$$

The parameter t_{12} comes out as

$$t_{12} = \frac{I_o}{\hat{I}_\pi} = \frac{\beta_F R_O [(R_s + R_B)(r_\pi + R_E) + R_s R_B]}{(R_C + R_O)[(R_s + R_B)(r_\pi + R_E) + R_s R_B] + R_E[(R_s + R_B)r_\pi + R_s R_B]}.$$

Then,

$$t_{22} = -\frac{T}{t_{12}} = -\frac{R_E (R_s + R_B)}{(R_s + R_B)(r_\pi + R_E) + R_s R_B},$$

which surprisingly is the same expression as for the basic common-emitter amplifier. The feedback factor and the gain factor are then also the same:

$$\beta = \frac{t_{22}}{t_{21}} = \frac{I_s}{I_o} = -\frac{R_E}{\frac{R_s R_B}{R_s + R_B}}$$

$$K = -\frac{1}{\beta} = -\frac{I_o}{I_s} = \frac{R_s || R_B}{R_E}. \quad (29)$$

This proves that the added R_O resistance does not add up to the feedback path as such. What is actually different are the return ratio T and the feed-forward term t_{11} , which has the value

$$t_{11} = \frac{R_s R_B R_E}{(R_C + R_O)[(R_s + R_B)(r_\pi + R_E) + R_s R_B] + R_E[(R_s + R_B)r_\pi + R_s R_B]}.$$

This equation is given in current mode and can be transformed easily to voltage mode as was done on the gain factor in the previous section covering the basic common-emitter amplifier. When the value of R_O is decreased, it affects both T and t_{11} , which lower the gain from the value of the ideal circuit, where r_o was assumed to be infinitely large.

The gain values obtained from these feedback parameters are in 100% agreement with values which are evaluated directly from the complete circuit. This means that the feedback analysis is still perfectly valid for these kind of circuits as well.

8.2 Input and output impedance

For the sake of completeness, the essential equations related to input and output impedances are listed here. The return ratio equations when the terminating impedances are short/open circuited are:

$$T(R_s \rightarrow 0) = \frac{g_m r_\pi R_O R_E R_B}{(R_O + R_C)[R_B(r_\pi + R_E)] + R_E R_B r_\pi}.$$

$$T(R_s \rightarrow \infty) = \frac{g_m r_\pi R_O R_E}{(R_O + R_C)[R_B + r_\pi + R_E] + R_E(R_B + r_\pi)}.$$

$$T(R_C \rightarrow 0) = \frac{g_m r_\pi R_O R_E (R_s + R_B)}{R_O[(R_s + R_B)(r_\pi + R_E) + R_s R_B] + R_E(R_B r_\pi + R_s r_\pi + R_s R_B)}.$$

The dead-system input impedance is

$$Z_{DI} = \frac{R_B(r_\pi + R_{Ex})}{R_B + r_\pi + R_{Ex}},$$

where

$$R_{Ex} = \frac{R_E(R_O + R_C)}{R_E + R_O + R_C}$$

and the dead system output impedance is

$$Z_{DI} = R_O + \frac{R_P R_E}{R_P + R_E},$$

where

$$R_P = r_\pi + \frac{R_s R_B}{R_s + R_B}.$$

These listed equations can be used for evaluating the total impedance using the Blackman's impedance formula. Against simulation results the gain and input impedance are exactly as obtained from these equations, but the output impedance has some difference in it when the value of R_O is taken very low. This does not mean that these equations derived from the small-signal model would not be correct, there just are other smaller details coming into play in the SPICE models when R_O is small.

9 HYPER BONUS: BUFFERED FEEDBACK CIRCUIT

Yet another example of a circuit which can be fully analysed using the general feedback method. The circuit is basically the same as the collector-to-base feedback, but this version introduces a unity-gain buffer in front of the actual gain stage. The circuit is (badly) drawn in Figure 38. Note that the biasing resistors for the buffer transistor have not been included here in the analysis.

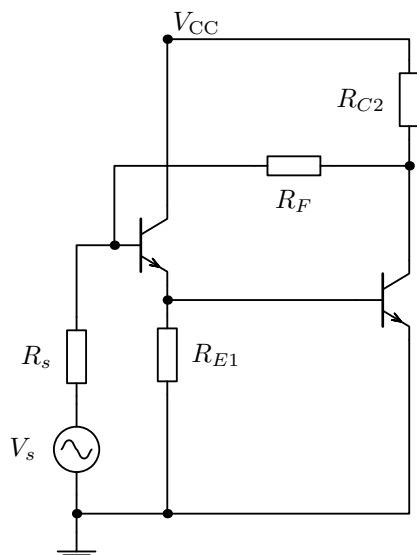


Figure 38: Buffered feedback circuit

The small-signal model of this circuit is shown in Figure 39.

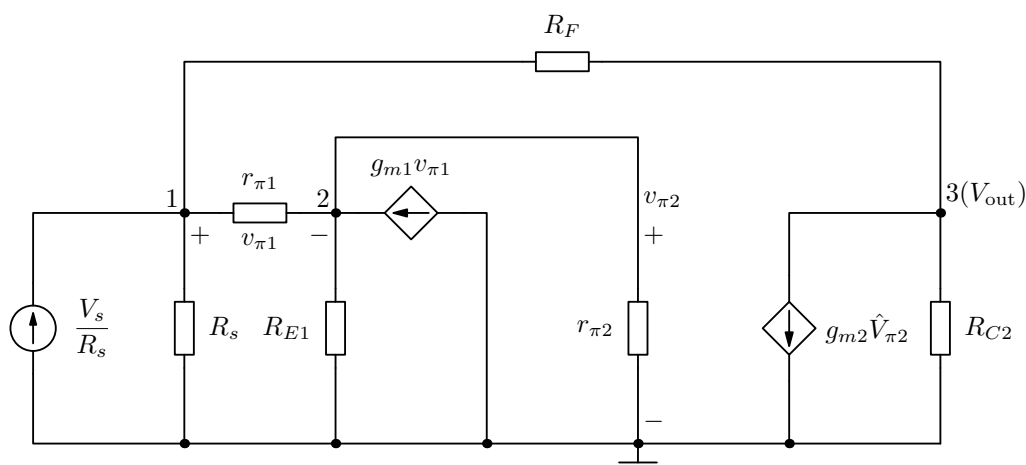


Figure 39: Small-signal model of the buffered feedback circuit

This circuit is evaluated quickly without excessive explanations, which can

be found from previous sections if needed. For this circuit we select $\hat{V}_{\pi 2}$ as the central controlled source used for analysis.

The t_{21} feedback parameter is evaluated from the circuit shown in Figure 40.

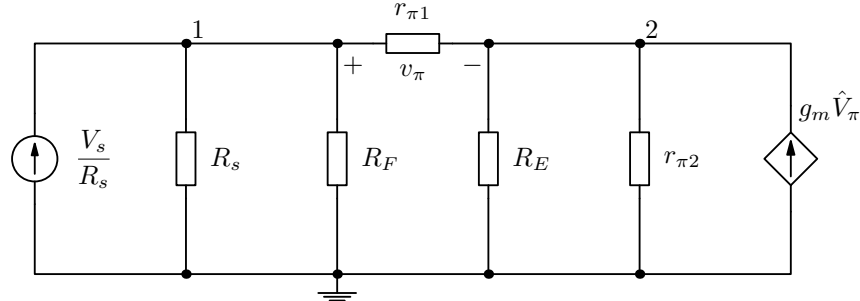


Figure 40: Small-signal model for evaluating feedback parameter t_{21}

The corresponding matrix equation, after moving the controlled source term into the admittance matrix, is

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_F} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_E} + \frac{1}{r_{\pi 2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ 0 \end{bmatrix}.$$

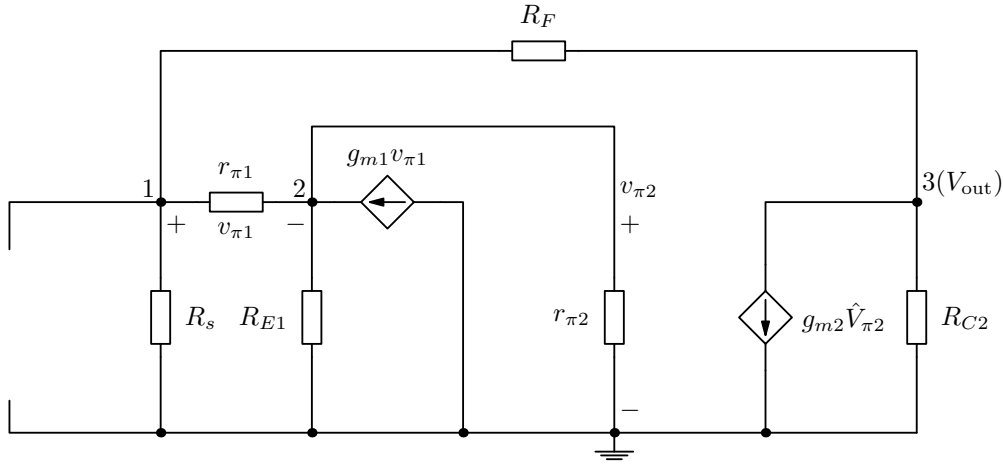
Here the feedback parameter t_{21} is solved for $V_2 = v_{\pi 2}$ as

$$t_{21} = \frac{v_{\pi 2}}{V_s} = \frac{R_F r_{\pi 2} (\beta_{F1} + 1) R_{E1}}{(R_s + R_F) [(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1}] + R_s R_F (r_{\pi 2} + R_{E1})},$$

because $\hat{V}_{\pi 2}$ was chosen as the base source for analysis.

Next we need a small-signal model to solve the return ratio T and the feedback factor t_{12} . This circuit is presented in Figure 41.

The return ratio T and the feedback factor t_{12} can be solved from the following matrix equation, which is obtained from Figure 41. The controlled source of g_{m1} has already been transferred into the admittance matrix and

Figure 41: Small-signal model for evaluating feedback parameters t_{12} and T

the base source g_{m2} is left into the current vector.

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_F} + \frac{1}{r_{\pi 1}} & -\frac{1}{r_{\pi 1}} & -\frac{1}{R_F} \\ -\frac{\beta_{F1} + 1}{r_{\pi 1}} & \frac{\beta_{F1} + 1}{r_{\pi 1}} + \frac{1}{R_E} + \frac{1}{r_{\pi 2}} & 0 \\ -\frac{1}{R_F} & 0 & \frac{1}{R_F} + \frac{1}{R_{C2}} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_{m2} \hat{V}_{\pi 2} \end{bmatrix}.$$

Using Cramer's rule, we get the following results for the needed feedback parameters:

$$T = -\frac{v_{\pi 2}}{\hat{V}_{\pi 2}} = \frac{\beta_{F2} R_{C2} R_s (\beta_{F1} + 1) R_{E1}}{(R_F + R_{C2})[(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1} + R_s r_{\pi 2} + R_s R_{E1}] + R_s [(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1}]}$$

$$t_{12} = \frac{V_{out}}{\hat{V}_{\pi 2}} = -\frac{\frac{\beta_{F2}}{r_{\pi 2}} R_{C2} [(R_s + R_F) [(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1}] + R_s R_F (r_{\pi 2} + R_{E1})]}{(R_F + R_{C2})[(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1} + R_s r_{\pi 2} + R_s R_{E1}] + R_s [(\beta_{F1} + 1) R_{E1} r_{\pi 2} + r_{\pi 1} r_{\pi 2} + r_{\pi 1} R_{E1}]}$$

From these equation it is possible to solve the gain factor

$$K = -\frac{R_F}{R_s},$$

which is the same as for the circuit without the buffer in front of the gain stage.

With these equations it is possible to form the voltage gain function as

$$A_F = A_v = \frac{t_{11} + KT}{1 + T},$$

where the feedforward parameter t_{11} is evaluated from circuit 42 using the same definitions as presented in the previous sections.

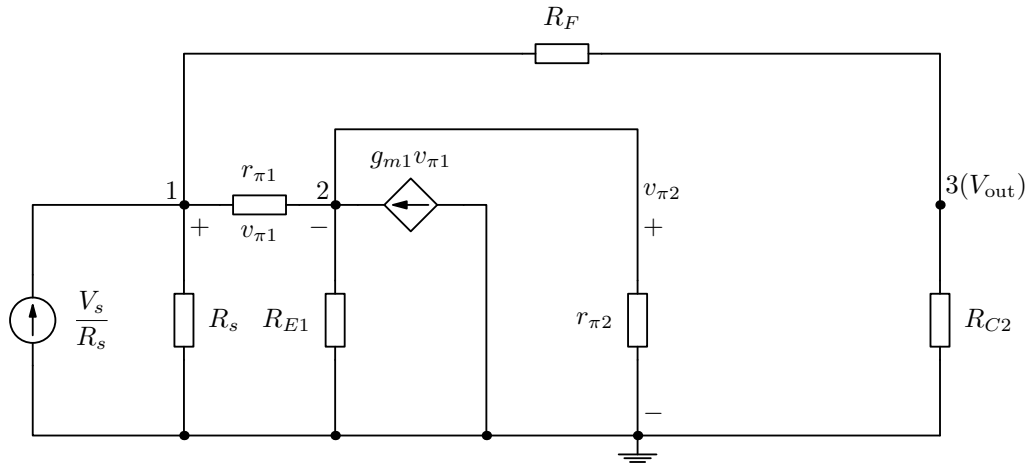


Figure 42: Small-signal model for evaluating feedforward parameter t_{11}

The impedance evaluation is omitted from this context, but it can be easily done based on the examples given in previous sections. When evaluating the dead-system impedance, remember to remove source g_{m2} and keep g_{m1} in the circuit to get the input impedance correctly.

Again, the extension to include R_{E2} in the evaluated feedback equations is achieved with the trick of assigning $r_{\pi 2} = r_{\pi 2} + (\beta_{F2} + 1)R_{E2}$. This trick is also explained better in the previous sections.

10 NOTES ABOUT BREAKING THE LOOP FOR LOOP GAIN EVALUATION

These notes are added after all the calculations have been made. There is the known rule that the loop can be broken at any point and the end result for the loop gain must be the same. During the analysis of the wah circuit, which uses the current-shunt configuration 2, I evaluated the loop gain twice, first breaking the loop at Q_1 and then at Q_2 . I was expecting to get the same result, but no, the results were different. Then I started thinking really hard what are the rules for breaking the loop correctly.

All the examples here use the controlled current source of the BJT as the break point of the loop. This is the most useful break point, because we don't need to add any extra terminating impedance, which should match the actual impedance at the break point. Concerning the current-shunt configuration, one can break the loop as presented, but that is the only way to use the controlled source as the break point and get the correct result. If we have an emitter resistor on Q_1 , we get two $\beta_{F1}\hat{I}_{\pi1}$ terms in the current vector. If we select to leave the first of these in the current vector and solve, we get a wrong answer. If we select to leave the second of these in the current vector and solve, we get the right answer. Why? I see two obvious reasons. When leaving the second current gain term in the current vector, we are not adding any new matrix elements in to the admittance matrix (the first gain terms add themselves into existing matrix elements). Secondly, there are no cross-matrix terms Y_{ij} on the row of the second current gain term, which directly indicates that there is no connection to the prior circuit elements, i.e. the loop is actually broken.

This leads to explain why the loop cannot be broken at Q_2 in the current-shunt configuration. If the first $\beta_{F2}\hat{I}_{\pi2}$ term on row 3 is transferred into the admittance matrix, second left at the current vector on row 4 and try to solve, there will be a column full of zeros in the admittance matrix when using Cramer's rule to solve the determinant quotient. If the second $\beta_{F2}\hat{I}_{\pi2}$ term on row 4 is transferred into the admittance matrix, first left at the current vector on row 3, then we are introducing additional matrix elements to the admittance matrix and this will lead to the incorrect result. So it appears that Q_2 is not actually within the loop in such a manner that it could be used for breaking it.

So all is clear up to now, we can use the controlled source as a break point if we actually notice it to break the connection to the preceding circuit elements and we are not adding any new matrix elements related to that controlled source which is used for solving the loop gain. This applies equally to situations where the BJT has or does not have an emitter resistor. For example, in the voltage-series configuration, one can break the loop on either Q_1 or Q_2 and get the same loop gain value.

But how about the examples of the shunt-shunt configuration and the emitter-follower? In the shunt-shunt configuration loop gain matrix, on row three there are no references to r_{π} (broken connection) and no new matrix ele-

ments are added. All is good.

The emitter follower configuration is a bit of a question mark. Now new matrix elements are added, but there is a reference to r_π on row two, which implies that the loop (what ever it might be in this case) is not broken. Maybe this is a special case where there is no loop at all.

So, as a main rule for loop gain evaluations by breaking the loop at the BJT controlled-source: Leave the single current gain factor into such a row, where there is no reference to preceding r_π and make sure that the possible other terms related to the same BJT are moved into the admittance matrix without adding any new matrix elements. These are the conclusions so far.

These thoughts also lead to believe that it is not always possible to break the loop at a controlled source. Then one must take the hard way and for example use the method where the loop gain is evaluated twice in open circuit and short circuit conditions and their reciprocal sum gives the loop gain. This method is very suitable when evaluating the loop gain with SPICE. More information on this method one can read from the Microelectronics book of Sedra and Smith, 6th edition.

11 OCTAVE SCRIPTS

```
% EXAMPLE OCTAVE SCRIPT FOR Current-Shunt AMPLIFIER
clear all
clc
% component value definitions
RS = 10000;
RC1 = 2000;
RC2 = 2000;
RE2 = 500;
RF = 10000;
rpi1 = 500;
rpi2 = 500;
B1 = 100;
B2 = 100;
gm = B1/rpi1;
B = 100;
RP = RS*rpi1/(RS+rpi1);

% t11 (dead system gain)
t11 = Io_Is = -(RS/RC2)*B2*rpi1*RE2*RC2 / ...
```

```

( (RS*rpi1+RS*RF+rpi1*RF)*(RE2*(B2+1)+rpi2+RC1)+RE2*(rpi2+RC1)*(rpi1+RS) )

% t21
t21 = Ii_Is = RS*(RE2+RF) / ( (RE2+RF)*(RS+rpi1)+RS*rpi1 )

% T (return ratio)
T = Ii_Ii = (RP/rpi1)*(B1*(B2+1)*RE2*RC1) / ( (RF+RE2+RP)*(rpi2+RC1)+(B2+1)*RE2*(RF+RP) )

% t12
t12 = Io_Ii = -(B1*B2*RC1)*(RE2+RF+RP) / ( (RF+RE2+RP)*(rpi2+RC1)+(B2+1)*RE2*(RF+RP) )

% t22
t22 = -T/t12
t22 = (RP/rpi1)*(B1*(B2+1)*RE2*RC1) / ((B1*B2*RC1)*(RE2+RF+RP))
t22_appr = RS*RE2 / ( (RE2+RF)*(RS+rpi1)+RS*rpi1 )

% BETA (reverse transmission factor)
BETA = t22/t21
BETA_appr = t22_appr/t21
BETA_appr = RE2/(RE2 + RF)

% K (gain factor)
K = -1/BETA
K_appr = -1/BETA_appr

% Total current gain
A_I = (t11 + K*T)/(1+T)

% T(Rs --> infinity)
T_ID = (B1*(B2+1)*RE2*RC1) / ( (RF+RE2+rpi1)*(rpi2+RC1) + (B2+1)*RE2*(RF+rpi1) )

% ZID and ZIF (dead-system input impedance and input impedance with feedback)
ZID = rpi1*( RF*(RE2*(B2+1)+rpi2+RC1)+RE2*(rpi1+RC1) ) / ...
      ( (rpi1+RF)*(RE2*(B2+1)+rpi2+RC1)+RE2*(rpi2+RC1) )
ZID_appr = rpi1*(RF+RE2) / (rpi1+RF+RE2)
ZIF = ZID/(1+T_ID)

% Z0 (output impedance with feedback) this simply equals RC2

% EXAMPLE OCTAVE SCRIPT FOR Voltage-Series AMPLIFIER
clear all
clc

% component value definitions
rpi1 = 5000;

```

```

B01 = 125;
rpi2 = 2500;
B02 = 125;
gm1 = B01/rpi1;
gm2 = B02/rpi2;
RC1 = 9000;
RC2 = 3000;
RE = 200;
RF = 6000;
RS = 2500;
RL1 = RC1*rpi2/(RC1+rpi2);
Rx = RE*(RF+RC2)/( RE+RF+RC2);

% t11 (dead system gain)
t11 = V0_Vs = RC2*(B01+1)*RE / ( (RC2+RF)*(RE*(B01+1)+rpi1+RS)+RE*(rpi1+RS) )

% t21
t21 = Vi_Vs = rpi1*(RF+RE) / ( (rpi1+RS)*(RE+RF)+(B01+1)*RE*RF )

% T (return ratio)
T = Vi_Vi = ( RE*(B01*RC1*B02*RC2)/(RC1+rpi2) )/( (RC2+RF+RE)*(rpi1+RS)+(B01+1)*RE*(RC2+RF) )

% t12
t12 = ( (B01*RC1*B02*RC2)/(RC1+rpi2) )*( ( (rpi1+RS)*(RE+RF)+(B01+1)*RE*RF )/rpi1 ) / ...
      ( (RC2+RF+RE)*(rpi1+RS)+(B01+1)*RE*(RC2+RF) )

% t22
t22 = -T/t12
t22 = -rpi1*RE/( (rpi1+RS)*(RE+RF)+(B01+1)*RE*RF )

% BETA (reverse transmission factor)
BETA = t22/t21
BETA = -RE/(RE + RF)

% K (gain factor)
K = -1/BETA

% Total voltage gain
Av = (t11 + K*T)/(1+T)

% T(Rs --> 0)
T_ID = ( RE*(B01*RC1*B02*RC2)/(RC1+rpi2) ) / ( (RC2+RF+RE)*rpi1+(B01+1)*RE*(RC2+RF) )

% ZID and ZIF (dead-system input impedance and input impedance with feedback)
ZID = rpi1 + ( (RC2+RF)*RE*(B01+1) / (RC2+RF+RE) )

```

```
ZIF = ZID*(1+T_ID)
```

```
% T(RC2 --> infinity)
```

```
T_OD = ( RE*(B01*RC1*B02)/(RC1+rpi2) ) / ( (rpi1+RS)+(B01+1)*RE )
```

```
% ZOD and ZOF (dead-system output impedance and output impedance with feedback)
```

```
ZOD = RF + ( RE*( (rpi1+RS)/(B01+1) ) / ( RE+( (rpi1+RS)/(B01+1) ) ) )
```

```
ZOF = ZOD/(1+T_OD)
```
