Equally spaced pipes inside an another pipe

December 23, 2012

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This is just a short example of a real life problem involving trigonometry. Once upon a time I was working in an engineering office which designed industrial ventilation systems. Without going too much into the details, the problem was to fit a large amount of small-diameter ventilation pipes inside one bigger pipe. One desire was to keep a constant distance between all the small pipes and the big pipe as well. The other desire was to maximise the number of smaller pipes inside the big pipe.

The 'solution' presented here solves the first demand, but not the second one since a 'squared' formation of the small pipes allowed to fit more pipes than the 'circled' formation, which is analysed in this context and presented in Figure 1. The notations used are: x is the smallest distance between the surface of each pipe, r is the radius of the smaller pipes and R_0 is the radius of the large pipe. Additionally Y_y and Y_x are distances of the 'measurement triangle' in the y- and x-coordinate directions respectively.

After a long while of pondering, I came up with the following 'design' equations to get exact measurements of any parameter x, r and R_0 if two of the other parameters are known. So, when the radius r of the smaller pipes and the radius R_0 of the bigger pipe are known, the smallest distance between each pipe surface is found from the equation

$$x = \frac{2\left[(R_0 - r)\sin\left(\frac{\pi}{n}\right) - r\right]}{1 + 2\sin\left(\frac{\pi}{n}\right)}. \tag{1}$$

When the smallest distance x between each pipe and the radius R_0 of the bigger pipe are known, the radius of the smaller pipes is found from the equation

$$r = \frac{2R_0\sin\left(\frac{\pi}{n}\right) - x\left[1 + 2\sin\left(\frac{\pi}{n}\right)\right]}{2\left[1 + \sin\left(\frac{\pi}{n}\right)\right]}. \tag{2}$$

And lastly, when the smallest distance x between each pipe and the radius r of the smaller pipes are known, the radius of the large pipe is found from the equation

$$R_0 = \frac{2r\left[1 + \sin\left(\frac{\pi}{n}\right)\right] + x\left[1 + 2\sin\left(\frac{\pi}{n}\right)\right]}{2\sin\left(\frac{\pi}{n}\right)}. \tag{3}$$

At this point one should ask the question, 'what is n in the previous equations?'. The parameter $n = 12$ is the amount of 'pipes' in the outermost

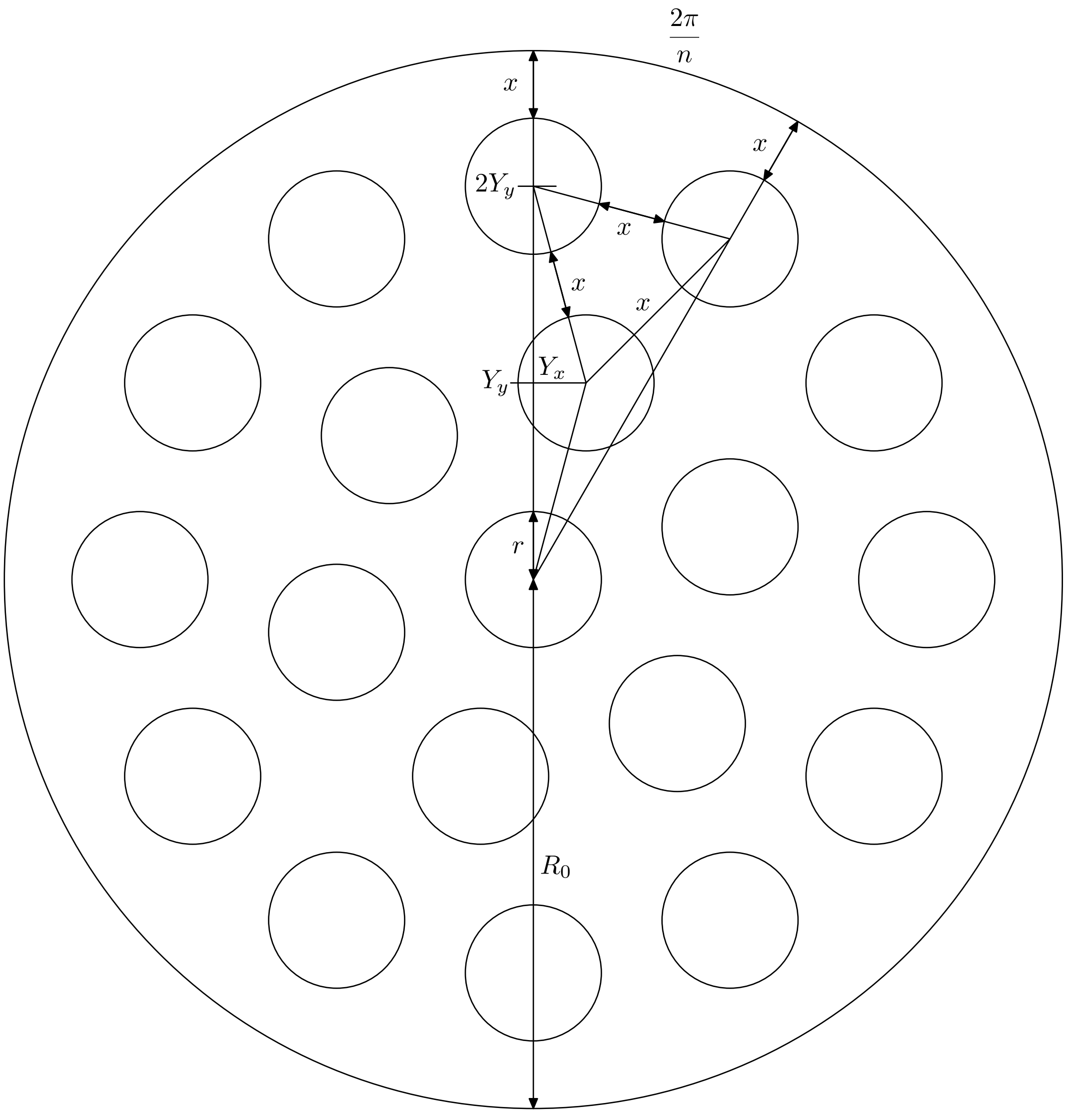

Figure 1: Smaller 'pipes' inside a large 'pipe'

circle. Unfortunately this design holds only for this specific configuration, so therefore n should always equal 12 and the number of pipes at the outermost circle should also be 12.

The equations for the 'measurement triangle' shown in Figure 1 are

$$Y_y = 2\left(r + \frac{x}{2}\right)\sin\left(\frac{\pi}{3} + \frac{\pi}{n}\right) \tag{4}$$

$$Y_x = 2\left(r + \frac{x}{2}\right)\cos\left(\frac{\pi}{3} + \frac{\pi}{n}\right). \tag{5}$$