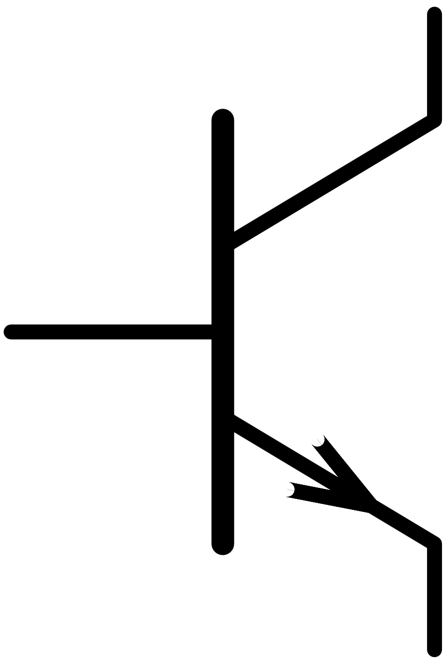
Calculating the voltage gain and output impedance of BJT amplifier stages



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This example shows how to evaluate the small-signal voltage gain and the output impedance of a bipolar junction transistor (BJT) amplifier stage. The output impedance will be evaluated using the results of Thévenin and Norton theorems. According to these theorems, the output impedance of any circuit stage is obtained as the quotient of open circuit voltage and short circuit current. This approach leads to a convenient example because in broader sense it demonstrates how to apply both nodal and mesh analysis methods to BJT circuits.

The analysed circuit contains the BJT device in a common-emitter configuration as shown in Figure 1. The analysis of other BJT amplifier configurations (common-collector and common-base) follow the same procedure as for the common-emitter amplifier.

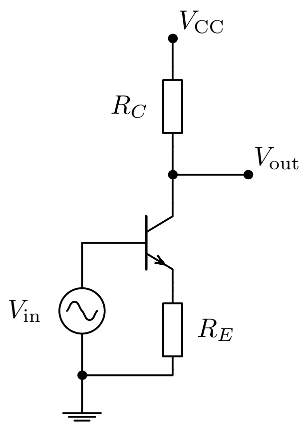
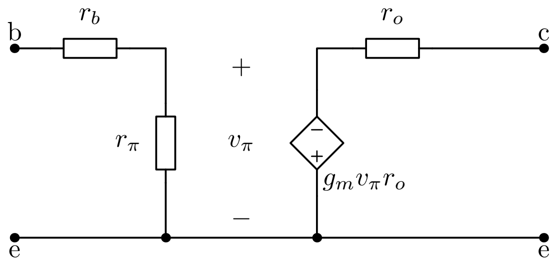
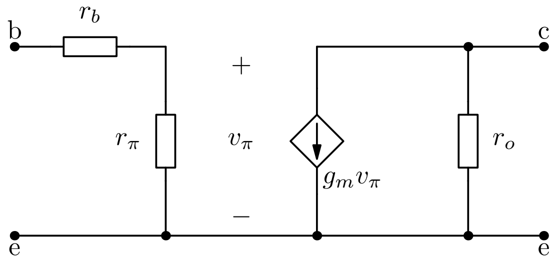


Figure 1: A BJT amplifier connected in a common-emitter configuration

When considering input signals of small amplitudes, the bipolar junction transistor is most often modelled as a linear current-controlled current source (CCCS). However, both the current-controlled voltage source (CCVS) and the current-controlled current source (CCCS) models are suitable to depict the BJT device, because the controlled source can be transformed accordingly using the Thévenin and Norton theorems of circuit analysis. Figure 2 indicates the CCCS and CCVS small-signal models for a general bipolar junction transistor. These models are applicable only for audio frequencies because the semiconductor junction capacitances have been neglected to simplify the following analysis. In a more realistic model, the semiconductor pn junctions behave as small capacitors that affect the high-frequency response of the BJT device. The linear small-signal gain of the BJT is modelled by the forward current gain factor $\beta_F = g_m r_\pi$, where the term g_m refers to the internal transconductance and r_π refers to the internal input resistance of the transistor. Quite often the small series resistance r_b is neglected in the analysis so that the input resistance of a BJT is solely determined by r_π.



(a) a voltage source model



(b) a current source model

Figure 2: Controlled source models for a BJT device

The nodal method of analysis is first used for calculating the voltage gain transfer function of the common-emitter amplifier stage. The rules of the nodal analysis method require that all alternating (signal) sources in the circuit are represented as current sources. This requirement can usually be filled by applying the source transformations defined by the Norton and Thévenin theorems. Additionally, all the static DC sources are considered to behave as the ground node from the viewpoint of alternating signals. This is why all the wires originally connected to DC sources are reconnected to the ground node in the small-signal equivalent circuit.

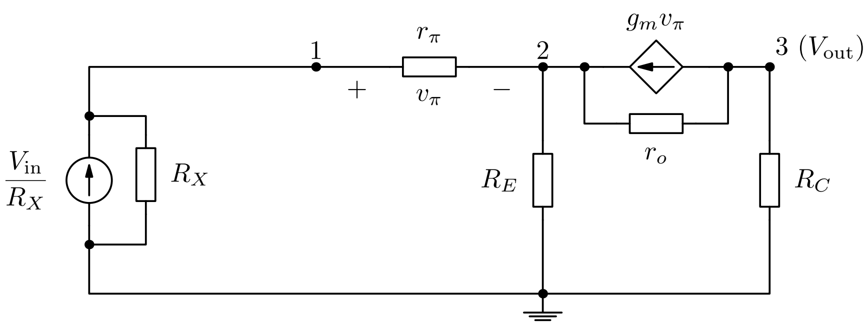


Figure 3: A voltage node model of the common-emitter BJT amplifier

Figure 3 illustrates the equivalent circuit of the common-emitter amplifier of Figure 1, which is modified according to the rules of nodal analysis. Compared to the circuit shown in Figure 1, this small-signal model contains an additional resistor R_X. This resistor depicts the internal resistance of the signal source V_{in}, which is required to transform the voltage source to a current source. The voltage nodes are indexed with numbers $1, 2$ and 3. The node 3 together with the ground node are the output terminals of the circuit. According to the rules of nodal analysis, the small-signal model of the common-emitter amplifier is represented by the matrix equation

$$\begin{bmatrix} \frac{1}{R_X} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{1}{r_\pi} & \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o} & -\frac{1}{r_o} \\ 0 & -\frac{1}{r_o} & \frac{1}{r_o} + \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_{\text{in}}}{R_X} \\ g_m v_\pi \\ -g_m v_\pi \end{bmatrix}.$$

This matrix equation can be simplified by noting that the control voltage v_π can be expressed as the difference of node voltages V_1 and V_2, namely $v_\pi = V_1 - V_2$. Based on this observation, the transconductance terms can be moved from the output current vector to the admittance matrix side. This little trick will make the symbolic evaluation of the circuit much simpler. It should be noted that also in matrix equations when terms are moved to the other side of the equal sign the term changes its sign from positive to negative or vice versa. After transferring the transconductance terms from the current vector to the admittance matrix, the final form of the matrix equation is

$$\begin{bmatrix} \frac{1}{R_X} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{\beta_F + 1}{r_\pi} & \frac{\beta_F + 1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o} & -\frac{1}{r_o} \\ \frac{\beta_F}{r_\pi} & -\frac{\beta_F}{r_\pi} - \frac{1}{r_o} & \frac{1}{r_o} + \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_{\text{in}}}{R_X} \\ 0 \\ 0 \end{bmatrix}.$$

The output voltage can be solved systematically using Cramer's rule. An application of this rule yields a determinant division

$$V_3 = V_{\text{out}} = \frac{\begin{vmatrix} \frac{1}{R_X} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & \frac{V_{\text{in}}}{R_X} \\ -\frac{\beta_F + 1}{r_\pi} & \frac{\beta_F + 1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o} & 0 \\ \frac{\beta_F}{r_\pi} & -\frac{\beta_F}{r_\pi} - \frac{1}{r_o} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_X} + \frac{1}{r_\pi} & -\frac{1}{r_\pi} & 0 \\ -\frac{\beta_F + 1}{r_\pi} & \frac{\beta_F + 1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o} & -\frac{1}{r_o} \\ \frac{\beta_F}{r_\pi} & -\frac{\beta_F}{r_\pi} - \frac{1}{r_o} & \frac{1}{r_o} + \frac{1}{R_C} \end{vmatrix}},$$

which can be evaluated in symbolic form by applying the basic steps of solving a determinant in the numerator and the denominator. The solution of the determinant quotient for the node voltage V_3, which is the output voltage V_{out}, is the transfer function

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-\beta_F R_C}{R_X + r_\pi + (\beta_F + 1)R_E + \frac{R_C(R_E + r_\pi + R_X)}{r_o} + \frac{R_E(r_\pi + R_X)}{r_o}}.$$

As the internal output resistance r_o of a BJT is considered very large, the fractional terms in the denominator are small and can be neglected in approximate calculations. Hence, a rough estimate of the output voltage is obtained from equation

$$V_3 = \frac{-V_{\text{in}}\beta_F R_C}{R_X + r_\pi + (\beta_F + 1)R_E}.$$

This result along with the short circuit current expression is later used for evaluating the formula for the output impedance of the common-emitter stage.

The analysis of the mesh currents requires its own small-signal equivalent circuit, which is obtained with slight modifications from the nodal analysis model. Basically all that is needed to reach the mesh-specific small-signal circuit is to transform the current sources to voltage sources. In many cases this approach is more convenient since the nodal analysis often forces to introduce the source resistance R_X from nowhere to be able to make the necessary signal source transforms. Figure 4 illustrates the redrawn common-emitter circuit suitable for mesh analysis.

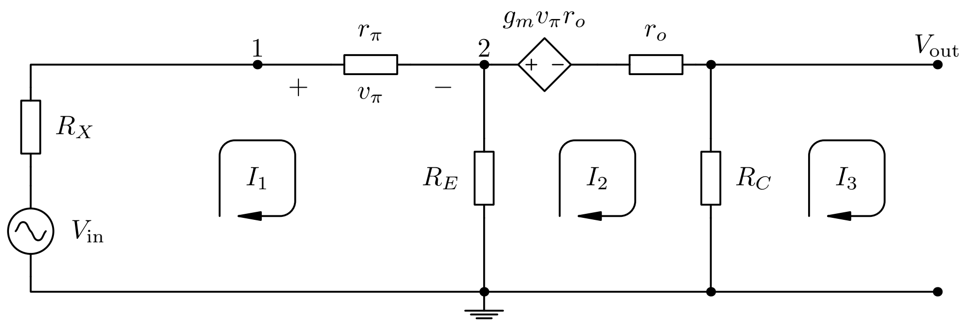


Figure 4: The mesh current model of the common-emitter amplifier

The output short circuit current is evaluated from the matrix equation

$$\begin{bmatrix} R_X + r_\pi + R_E & -R_E & 0 \\ -R_E & R_E + r_o + R_C & -R_C \\ 0 & -R_C & R_C \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{\mathrm{in}} \\ -g_m v_\pi r_o \\ 0 \end{bmatrix}.$$

The voltage v_π in this matrix equation can be expressed using the mesh currents as $v_\pi = I_1 r_\pi$, and therefore the mesh-matrix is reshaped to

$$\begin{bmatrix} R_X + r_\pi + R_E & -R_E & 0 \\ -R_E + \beta_F r_o & R_E + r_o + R_C & -R_C \\ 0 & -R_C & R_C \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{\mathrm{in}} \\ 0 \\ 0 \end{bmatrix}.$$

The use of Cramer's rule for solving the output short circuit current I_3 leads to a solution

$$I_3 = \frac{V_{\mathrm{in}} \left(-\beta_F + \dfrac{R_E}{r_o} \right)}{R_X + r_\pi + (\beta_F + 1) R_E + \dfrac{R_E (R_X + r_\pi)}{r_o}}.$$

Finally, the output impedance is evaluated as the quotient of $\frac{V_3}{I_3}$, which equals

$$\frac{-\beta_F R_C \left(R_X + r_\pi + (\beta_F + 1) R_E + \dfrac{R_E (R_X + r_\pi)}{r_o} \right)}{\left(-\beta_F + \dfrac{R_E}{r_o} \right) \left(R_X + r_\pi + (\beta_F + 1) R_E + \dfrac{R_C (R_E + r_\pi + R_X)}{r_o} + \dfrac{R_E (r_\pi + R_X)}{r_o} \right)}.$$

When considering the internal output resistance r_o to be infinitely large, the output impedance of the common-emitter BJT amplifier configuration is simply R_C.