A case study: re-engineering the Big Muff π

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After learning a lot on traditional transistor biasing techniques and general analysis methods of feedback amplifiers it is time to extend these skills to something more exotic than textbook examples. After a few days of trying to figure out what is actually happening in the good old Big Muff π effect circuit this text just needed to be written to document the findings and the results of various mathematical evaluations.

The Big Muff π is a classic distortion pedal from the late 1960's. Due to its popularity, some modified and improved versions of the Big Muff π have been released from time to time until the recent years. The 'original' Big Muff π effect circuit consists of an input gain stage, two clipping stages, a tone control section and an output gain stage. The circuit uses basic transistors in a very fruitful manner, for us all to learn something new. Figure 1 depicts the overall view of the circuit schematic. The schematic has been populated with component values from the early versions of the Big Muff π.

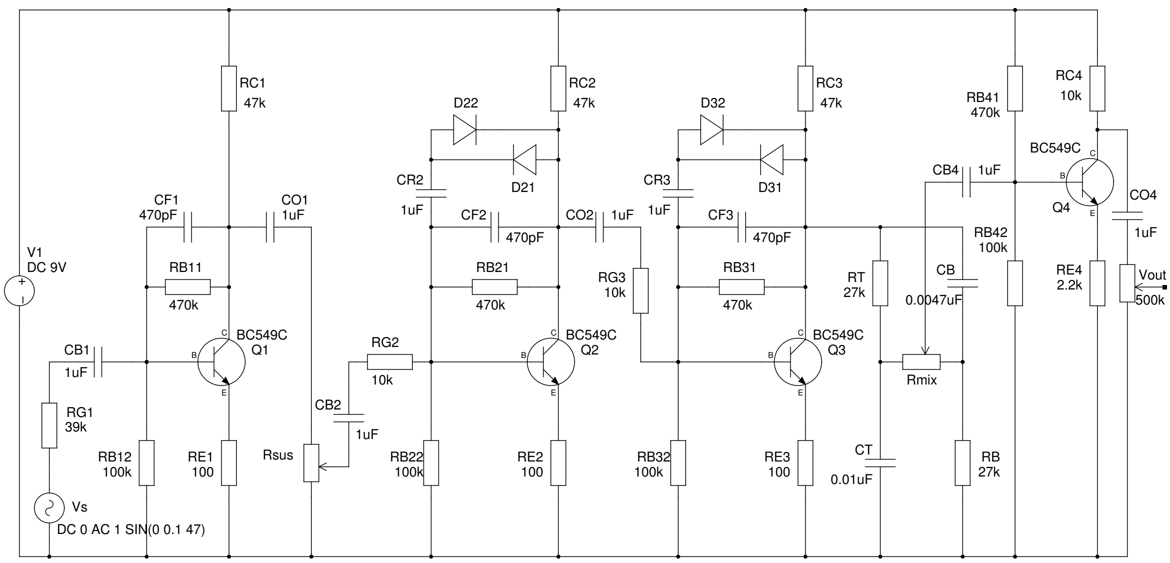


Figure 1: The Big Muff π circuit diagram

This document tries to explain how and why the designer has chosen the shown component values for the given circuit configuration. What we really cannot know is, why the designer has chosen to use these circuit blocks, but that is more like artistic freedom for the designer to choose the circuit configuration to play with. The main idea is to learn mathematical design methods mostly by analysing the existing circuit configurations and finding the key components acting on the specific features of each stage of the circuit. The derived equations or detailed explanations should help to choose correct component values and to understand which components affect to certain properties of analogue signal processing.

The first three sub-stages of the circuit all use a similar gain configuration, which heavily builds upon the negative feedback from the collector to the base of the transistor. It is better to utilise the feedback analysis methods to this stage first to get a better grasp on what affects the gain and the impedance of this configuration.

1 Gain stage small-signal design and analysis

The purpose of the first gain stage is to act as a preamplifier, which can be used for controlling the sustain and set a suitable signal level for the following stages. The similar BJT structure is used also in the two clipping stages, so it is even more beneficial to analyse the gain configuration properly.

To make life easier in the gain and impedance analysis, let's use a simplified gain stage circuit, where the small emitter resistance is neglected and the DC-blocking capacitor is considered to be a short-circuit at the analysis frequency. Also the feedback capacitor is removed, but that can be included later as a simple replacement where R_{B1} is replaced by Z_{B1}. Figure 2 visualises the simplified circuit which we are dealing with.

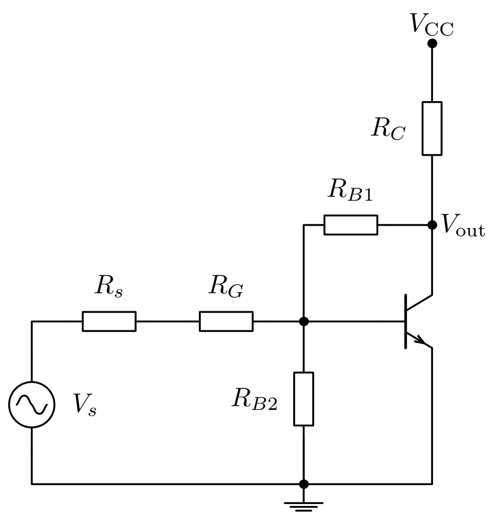


Figure 2: Simplified mid-frequency gain stage with signal source included

As seen from Figure 2, the signal source V_s and its internal resistance R_s are also included in the analysis. According to the Norton and Thevenin theorems, the signal source can equally be drawn as a current source in parallel with R_s to better represent the guitar pickup as a signal source. The signal source can also depict a preceding gain stage in series with its output impedance R_s.

Figure 3 shows a small-signal model based on the simplified gain stage of the Big Muff π circuit shown in Figure 2. As stated above, the simplifications concern the removal of DC-blocking capacitors (evaluation is therefore done in mid-frequency range), removal of R_E because it has such a small value, and R_{B2} in parallel with r_π effectively equals r_π. Later it is possible to include R_{B2} as a parallel resistance into r_π, but then it is important to use $g_m r_\pi$ instead of β_F in the following equations. The resistor R_{B1} has been renamed to R_2 in the small signal model, because we are expecting it to have some connection to R_1, which in turn is renamed from R_G. These renamings have been done because the configuration is awfully similar to the non-inverting op-amp circuit and we are expecting a comparable result.

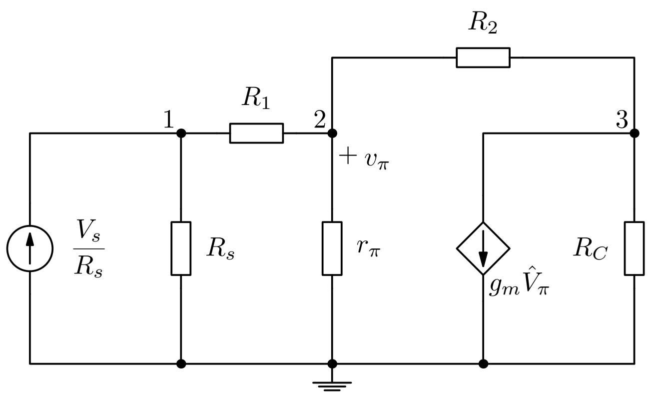


Figure 3: Small-signal model of the simplified gain stage at mid-frequencies

Anyway, let's analyse the basic feedback parameters, the feedback factor β and the return ratio T, for this simplified small-signal circuit. The analysis follows the general feedback analysis method, which is explained in detail in the other text titled: 'Basic Forms Of Feedback in Voltage and Current Amplifiers'. According to the standard procedure, first we calculate the feedback parameter t_{21}, which requires that the output voltage node is set to zero before evaluation. This leads to the following matrix equation:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_1} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_\pi} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_s} \\ 0 \end{bmatrix} \tag{1}$$

Solving for the node voltage V_2, this gives:

$$t_{21} = \frac{V_2}{V_s} = \frac{R_2 r_\pi}{r_\pi(R_s + R_1 + R_2) + R_2(R_s + R_1)}. \tag{2}$$

Next we evaluate the return ratio T, which requires the input source to be set to zero. When a current source is zero, it leaves an open circuit and from that configuration the following matrix equation is obtained:

$$\begin{bmatrix} \frac{1}{R_s} + \frac{1}{R_1} & -\frac{1}{R_1} & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_\pi} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_C} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_m \hat{V}_\pi \end{bmatrix}. \tag{3}$$

After solving for V_2, the equation for the return ratio T is found to be

$$T = -\frac{V_2}{\hat{V}_\pi} = \frac{g_m r_\pi R_C(R_s + R_1)}{(R_s + R_1)(R_2 + R_C + r_\pi) + r_\pi(R_2 + R_C)}. \tag{4}$$

One can use the same matrix equation to solve for node voltage V_3 and get the feedback parameter t_{12}, which comes out as

$$t_{12} = \frac{V_3}{\hat{V}_\pi} = -\frac{g_m R_C[r_\pi(R_s + R_1 + R_2) + R_2(R_s + R_1)]}{(R_s + R_1)(R_2 + R_C + r_\pi) + r_\pi(R_2 + R_C)}. \tag{5}$$

This parameter can be used for evaluating the open-loop gain, which is approximately the product $t_{12}t_{21}$. However, in this context we don't really care about it. More importantly the reverse transmission transfer function (or the feedback factor) β can now be evaluated, but we still need the parameter t_{22} for that. One gets the missing parameter from

$$t_{22} = -\frac{T}{t_{12}} = \frac{r_\pi(R_s + R_1)}{r_\pi(R_s + R_1 + R_2) + R_2(R_s + R_1)}, \tag{6}$$

and then

$$\beta = \frac{t_{22}}{t_{21}} = \frac{R_s + R_1}{R_2} \quad \Rightarrow \quad K = -\frac{R_2}{R_s + R_1}. \tag{7}$$

The gain factor K is the negative inverse of the feedback factor by definition, meaning that

$$K = -\frac{1}{\beta}. \tag{8}$$

It is important to note that the since the feedback evaluation was completely done using voltage signals $\frac{V_x}{V_y}$, this gain parameter is valid only for describing voltage gain.

According to the general theory of feedback, if the forward transmission path is neglected, the total gain obtained from the circuit with feedback is approximately defined by the return ratio T and the voltage gain factor K as

$$A_F \approx \frac{KT}{1+T}. \tag{9}$$

For further evaluations is is beneficial to simplify the exact equation for the return ratio. After pulling a few common factors out and leaving out r_π on the basis of it being much smaller than $R_1 + R_C$, one finds that the return ratio

$$T \approx \frac{g_m r_\pi}{\left(1 + \frac{r_\pi}{R_s + R_1}\right)\left(1 + \frac{R_2}{R_C}\right)}. \tag{10}$$

Alongside with K, this equation can be used to approximate the gain of the given circuit stage. Basically if $T > 10$, then it is good enough approximation to say that the gain

$$A_F \approx K. \tag{11}$$

This approximation can be used in the gain stages of the Big Muff π.

And one more thing about the gain. Let's think a little further about the gain equation

$$K = -\frac{R_2}{R_s + R_1}. \tag{12}$$

In the Big Muff circuit R_2 is parallel with a capacitor, let's call it C_2 to match with R_2. This parallel capacitor makes the gain frequency dependent. Luckily we do not need to calculate everything again, just replace R_2 with

$$Z_2 = \frac{R_2}{1 + 2\pi f C_2 R_2}, \tag{13}$$

where f refers to the frequency. Then we have

$$K = -\frac{Z_2}{R_s + R_1}, \tag{14}$$

and according to simple numerical evaluation, with standard component values, the impedance Z_2 is about $\frac{R_2}{2}$ at 1 kHz and $\frac{R_2}{15}$ at 10 kHz. So the amplification of the Big Muff gain configuration decreases significantly for high frequencies.

The input and output impedances are evaluated using the Blackman's impedance formula. Regarding the input impedance, the Blackman's formula leads to the conclusion that the input impedance with feedback is practically the same as the dead system input impedance, which is dominated by R_1 (since R_s is not considered to be part of the feedback impedance).

However the output impedance is scaled down considerably according to the Blackman's formula. The dead system output impedance is approximately R_2 in series with a smaller parallel impedance. Therefore, a suitable approximation for the output impedance is just R_2. Still one needs to evaluate the return ratio in the open-circuited and short-circuited conditions. The short-circuit condition leads to a zero value, but the open-circuit condition gives a reasonable value:

$$T(R_C \to \infty) \approx \frac{g_m r_\pi}{1 + \dfrac{r_\pi}{R_s + R_1}}.$$

Hence the use of the Blackman's impedance formula for the output impedance with feedback gives

$$Z_{OF} = \frac{Z_{OD}}{1 + T(R_C \to \infty)} \approx \frac{R_2}{1 + T(R_C \to \infty)}.$$

However, since this feedback output impedance is parallel with R_C, the output impedance when looking into the gain stage *after* R_C is $R_C || Z_{OF}$, the smaller of which would be the dominating impedance. This output impedance will also be the input source impedance Z_s for the following gain stage.

When using the shunt-shunt feedback topology, the amplifier becomes a transresistance amplifier, which expects current to the input side and amplifies this current into a voltage output signal. Since the input signal expects current, the effect device works best when connected directly to the guitar output, which can be considered mainly as a current signal.

Ok, now we know what components affect the gain and the interfacing impedances. Because the gain is determined by the feedback resistor which provides also DC bias to this circuit, one must take this into account when designing the biasing for the gain stages of the Big Muff π.

2 Gain stage bias design and analysis

So next the biasing scheme, how to design and how to analyse it? Figure 4 shows the basic DC configurations, one with the emitter resistor included and one without it. It is better to attack the design first and then verify the results by doing the analytic analysis on the obtained circuit.

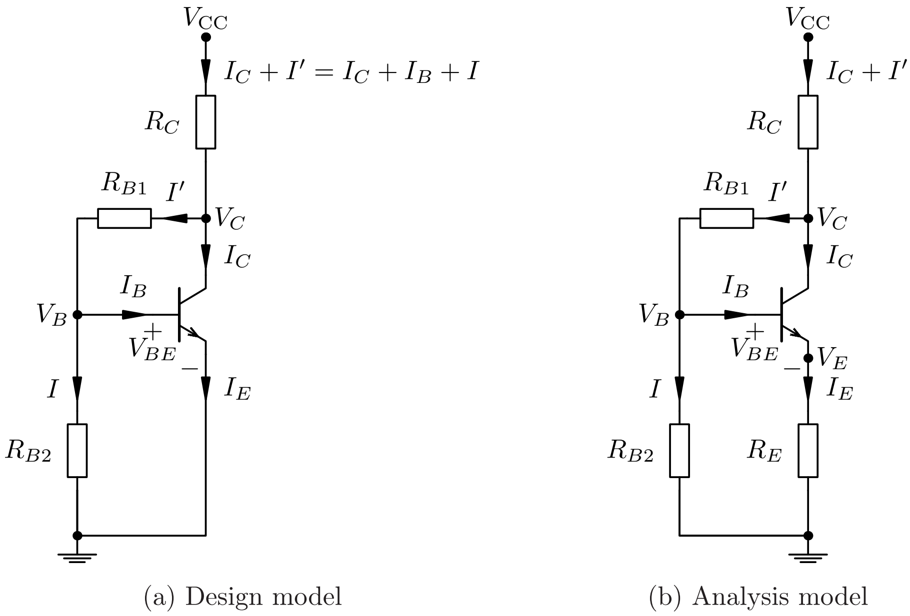


Figure 4: DC models for design and analysis

Almost always the biasing design begins by choosing the collector resistor. In this biasing scheme, the collector resistor mainly just sets a limiting value for the collector and emitter current. The gain and output impedance are mainly determined by the feedback configuration. So let's not suck too much juice from the battery and select $R_C = 47$ kilohms. As noted in the small-signal analysis, the output impedance will not be equal to R_C unless the value if R_C is chosen to be relatively small. If we aim for collector bias voltage $V_C \approx V_{CC}/2$, then the collector current with $R_C = 47$k is around 0.1 mA.

The base bias network is almost like the typical shown in every textbook, but now the feedback from the collector is messing up everything. Luckily the same design rule applies to both biasing configurations: the main design rule in BJT biasing is that the current flowing in the base bias divider network should be approximately 1/10 of the collector current. This follows from the approximation that taking 1/10 parts from the collector current allows the collector bias voltage to be approximated without the base bias divider current. On the other hand, if $\beta \approx 100$, the base current I_B is again 1/10 from the base bias divider current. On the same argument as before, 1/10 of base bias divider current can be neglected in approximations, so the design is simplified by making these approximations.

Now when we assume that the emitter is tied to ground (valid even with small emitter resistors since R_C limits the emitter current to be quite small), we assume that $V_B = V_{BE} \approx 0.65$ volts (the base-emitter voltage might actually be somewhere between 0.45 V - 0.75 V). When placing a certain-sized resistor between V_B and ground, it will start demanding current based on Ohm's law. Since the transistor's base cannot provide the current, we must use another biasing resistor R_{B1} to source that current from the collector. So to begin with, we can select R_{B2} so that

$$\frac{V_{BE}}{R_{B2}} \approx \frac{I_C}{10}. \tag{15}$$

Hence,

$$R_{B2} \approx \frac{10 V_{BE} R_C}{V_{CC} - V_C} \tag{16}$$

and

$$R_{B1} \approx \frac{V_C - V_{BE}}{V_{BE}} R_{B2} \tag{17}$$

to set the voltage divider approximately to V_{BE} with the given current. This is reasonably valid as long as the base current is much smaller than the current flowing in the base divider. The rounding of the resistance values should be done so that R_{B1} is rounded downwards and R_{B2} is rounded upwards to the nearest standard value. When we chose $R_C = 47$ kΩ to limit the collector current to 0.1 mA, we now get $R_{B2} = 68$ kΩ and $R_{B1} = 390$ kΩ.

So what role does the transistor selection play here? Basically the equations (or the approximations made in the design) only say that the current gain factor β_F should be 100 or slightly more, so in practice a current gain of around 200 is suitable. Very high gain transistors ($\beta_F > 500$) might bring the bias point a little lower than 4 volts. Therefore one can choose the transistors quite freely, the low-cost 2N3904 is fine, I used BC549B, but it does not really have much difference to the affordable 2N3904 model.

However, one does not need to settle for the values obtained by this method. Basically only the ratio of the base resistors is meaningful in the biasing design, but one must keep in mind that when increasing the base bias resistances, the base current I_B becomes more and more prominent and it cannot be neglected in the design anymore. The value of R_{B1} can be for example chosen according to gain requirements, but the input impedance is determined mainly by the series resistance R_G, which is hiding behind the capacitor and does not affect the transistor biasing at all.

Next we derive some analysis equations for the general case with the emitter resistor included. The emitter resistor adds a bit more stability, and when kept small, it does not affect much on the collector feedback biasing scheme. According to the Kirchhoff's current rule, the current equation for the voltage node V_B is:

$$I' = I + I_B = I + \frac{I_E}{\beta_F + 1}, \tag{18}$$

and for the voltage node V_C:

$$I'_C = I_C + I' = I_E \frac{\beta}{\beta_F + 1} + I'. \tag{19}$$

After the current equations are written down for each voltage node, the next step is to express the currents using the supply voltage and the node voltages. The currents appearing in the current equations can be expressed with respect to the node voltage as:

$$I'_C = \frac{V_{CC} - V_C}{R_C} \quad ; \quad I_E = \frac{V_B - V_{BE}}{R_E} \quad ; \quad I' = \frac{V_C - V_B}{R_{B1}} \quad ; \quad I = \frac{V_B}{R_{B2}}$$

and after substituting these voltage equations to the current equations,

$$\begin{aligned} \frac{V_{CC} - V_B}{R_C} &= \frac{V_B - V_{BE}}{R_E} \frac{\beta}{\beta + 1} + \frac{V_C - V_B}{R_{B1}} \\ \frac{V_C - V_B}{R_{B1}} &= \frac{V_B}{R_{B2}} + \frac{V_B - V_{BE}}{R_E} \frac{1}{\beta + 1}. \end{aligned}$$

These two equations can be rearranged into a matrix equation from where the node voltages can be solved systematically:

$$\begin{bmatrix} \frac{\beta}{\beta+1}\frac{1}{R_C} + \frac{1}{R_{B1}} & \frac{\beta}{\beta+1}\frac{1}{R_E} - \frac{1}{R_{B1}} \\ -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{\beta+1}\frac{1}{R_E} \end{bmatrix} \times \begin{bmatrix} V_C \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{V_{CC}}{R_C} + \frac{\beta}{\beta+1}\frac{V_{BE}}{R_E} \\ \frac{1}{\beta+1}\frac{V_{BE}}{R_E} \end{bmatrix}.$$

The analytic expression to verify the collector voltage with the chosen bias values is:

$$V_C = \frac{\frac{V_{CC}}{R_C}\left(\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{\beta+1}\frac{1}{R_E}\right) + \frac{V_{BE}}{R_E}\left(\frac{1}{R_{B1}} + \frac{\beta}{\beta+1}\frac{1}{R_{B2}}\right)}{\frac{\beta}{\beta+1}\frac{1}{R_C}\left(\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{\beta+1}\frac{1}{R_E}\right) + \frac{1}{R_{B1}}\left(\frac{1}{R_{B2}} + \frac{1}{R_E}\right)}. \tag{20}$$

3 Clipping stage analysis

The purpose of the clipping stage(s) is to cut off the top and bottom parts of the input signal to introduce rich harmonic content to the sound. The magic is all coming from the oppositely connected diodes, but how to analyse them? Let's state the rules for small signal analysis for diodes. Firstly one needs to identify if the diodes are on or off (forward- or reverse-biased). Then one can refer to Figure 5 for which equivalent circuit to use. Diode is forward-

(a) Forward bias (b) Reverse bias

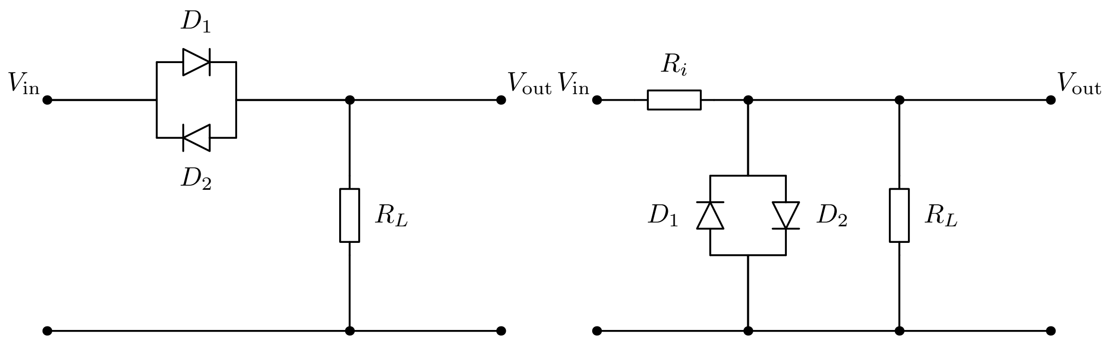
Figure 5: Diode equivalent circuits for analysis

biased if the voltage at the anode (A) is more than V_d above the cathode (K). Typically $V_d \approx 0.65$ volts is the same as the V_{BE} intrinsic base-emitter voltage in transistors. When a diode is forward-biased, its resistance r_d is very low (it can be considered a short-circuit which passes all current through it) and at the same time it has the almost constant voltage V_d acting over it.

When the anode voltage drops below V_d, the resistance increases tremendously and it can be considered an open circuit. So in a nutshell, the diode swaps states between open-circuit and short-circuit conditions depending on the external voltage laid across it.

A clipper circuit is a typical diode application in all fields of electronics. Diode clippers can be categorised in two main classes: series clippers and parallel clippers. Figure 6 explains the idea. In a static configuration without any input signal both diodes are off (open circuited), but there exists a voltage V_d over the diodes. To get a better grasp what is going on, imagine an input voltage, which swings between $-2V_d$ and $+2V_d$ volts. When the input signal exceeds V_d, the other diode starts to conduct. When the input signal drops below V_d, both diodes are off again. When the input signal drops below $-V_d$, the other diode starts to conduct. This statement is valid for both series and parallel diode pairs.

Depending if it is a series or parallel clipper the middle part or the top and bottom part of the input signal are clipped away. Figure 7 explains the idea.



(a) Series clipper (b) Parallel clipper

Figure 6: Series and parallel clipping stages

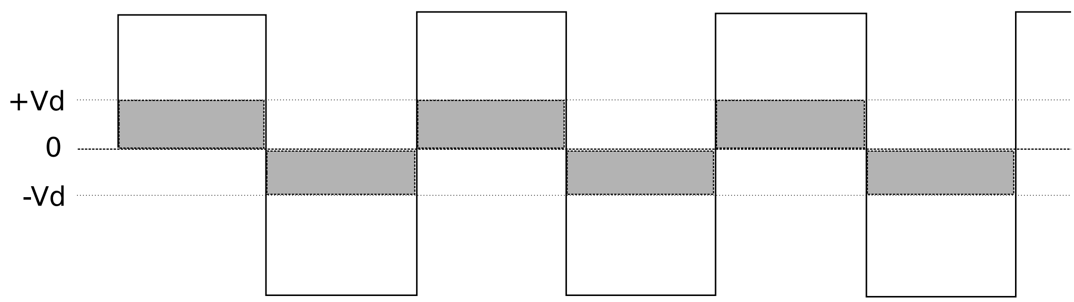


Figure 7: Diode clipping applied to a square wave

The series clipper passes the white areas of the square wave (top and bottom part) to the load and clips out the middle part. While the input signal increases above V_d, the other diode conducts and it pulls the anode potential up with the input signal. On the same time cathode voltage follows the changes of the anode potential at $V_{\text{in}} - V_d$, and this voltage is seen over the load resistor R_L. Within the grey region ($+V_d$ to $-V_d$) the diodes are open circuits (infinite resistance).

The parallel clipper passes the grey areas of the square wave (middle part) to the load and clips out the top and bottom. Here the diodes are tied to ground so that the input signal cannot pull these along. When the input signal exceeds the diode voltage limits $+V_d$ and $-V_d$, the signal is directed to ground, otherwise the diodes are open circuits and the signal passes directly to the load.

Now we should have all the necessary information to understand the Big Muff clipping stage. Many distortion effects utilise the parallel clipper, because it is so simple to set up. Just connect two diodes parallel to the output in opposite directions. The Big Muff has the clipper placed in the feedback loop of an amplifier section, and the idea is to clip the gain instead of the actual signal. Actually it is more descriptive to say that the diodes act as a gain limiter. And we should all agree now that Big Muff uses a *series* clipper in the feedback path.

How does it work then? We all would like to say it is simple, but it is not. The diodes are connected behind a capacitor, which blocks the DC flowing between the base and collector, but still allows the AC signal from the base to interact with the diodes. Let's go through step-by-step what happens after the battery is connected to the Big Muff circuit. The initial state is that the capacitor has discharged completely and measures zero volts across its terminals.

1. Battery is connected, transistor tries to balance into bias state. Collector voltage starts rising towards collector bias voltage V_C.
2. Capacitor sits on top of the base voltage, about V_{BE} that is. The rising collector voltage starts pulling the diodes towards V_C, the collector bias voltage.
3. The diode pointing from right to left starts to conduct and charges the capacitor through R_C. The capacitor voltage now follows the rising collector voltage, but lags the amount of V_d behind.
4. Collector reaches its bias voltage V_C, diode stabilises into steady-state mode (in the verge to conduct). Capacitor is at voltage $V_C - V_d$.

So, in a static configuration the DC voltage between the capacitor and the diodes is $V_C - V_d$. Since the capacitor blocks DC, but passes AC, the diodes are now fooled to believe that the signal at the transistor's base fluctuates around $V_C - V_d$. After the first gain stage, the signal level at the base of the clipping amplifier is about a volt or so, although depending heavily on the style of playing and sustain control of course. When the guitar signal is applied, base voltage at the clipper amplifier goes down, say -0.5 V and the voltage between the capacitor and the diodes is now $V_C - V_d - 0.5$ V. On the same time the gain of the amplifier makes the collector voltage shoot up. Now the voltage across the diodes is clearly over V_d and the diode pointing to the left is conducting.

When the diode conducts, the resistance of the feedback path drops to zero.

This bypasses the transistor completely and the gain drops to zero. Also the collector voltage is limited by the diode intrinsic voltage, so that in the beginning it will not exceed V_C. However, although the collector voltage appears to stay clipped, in the background there is a continuous battle between transistor gain and diode clipping, both in the verge of activating and deactivating in a fast pace.

Same pattern repeats when the signal at the transistor base goes positive. Then the collector swings to negative direction, diode pointing to the right starts to conduct and limits the collector voltage to $V_C - 2V_d$. Fight between gain and clipping continues until polarity is reversed again.

One more thing. Things are not in balance yet. The collector voltage is not swinging equally around the diode limiting voltages, since it needs to go $2V_d$ lower to limit the other direction. This will stabilise naturally into having oscillation limits $V_C + V_d$ and $V_C - V_d$. This is seen in the oscilloscope screen capture in Figure 8. While the diodes are opening and closing in the

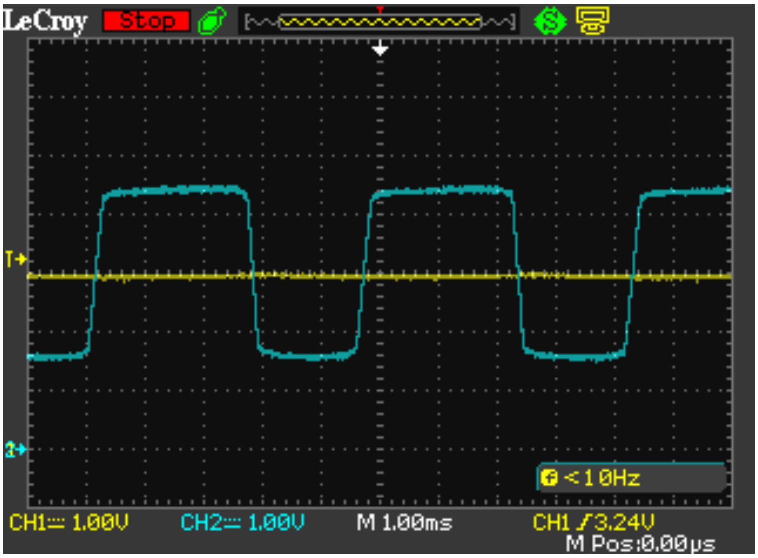


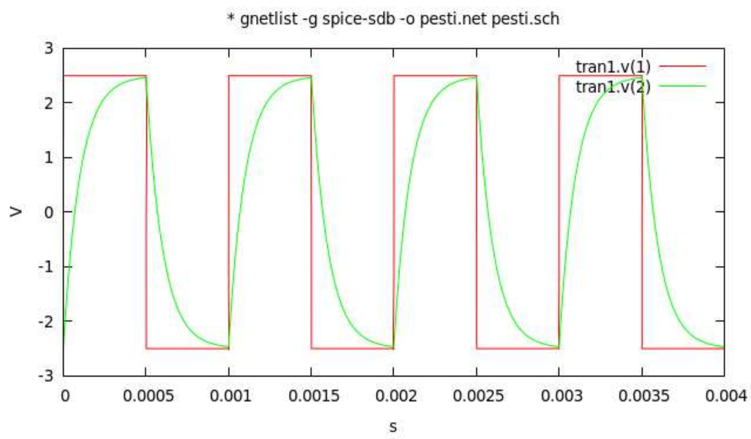
Figure 8: Clipping stage collector voltage (blue trace) and capacitor voltage (yellow trace) in the clipping stage under sinusoidal input signal

unbalanced state, the capacitor is partially charging through R_C. Eventually the balance is found when the capacitor voltage reaches V_C. When no signal is applied, the capacitor slowly discharges to the steady-state voltage of $V_C - V_d$.

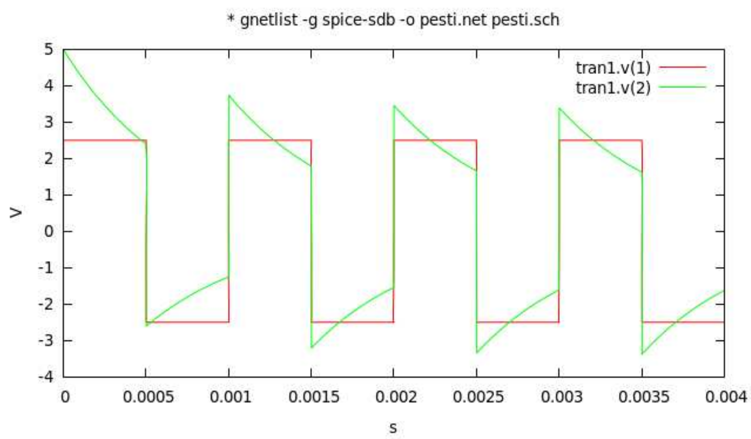
4 Tone stage design and analysis

The Big Muff π tone control stage is a classic. It is simple, yet effective. By taking one first order high-pass filter and one first order low-pass filter and combining them together by a mixing resistor, something amazing happens. Let's take a closer look at it to understand what exactly happens.

The output signal coming from the clipping stages is resembling a square wave with a peak-to-peak voltage of two times the diode threshold voltage, i.e. $V_{\mathrm{p-p}} \approx 2V_d$. How does low-pass and high-pass filter affect a square wave in general? Figure 9 gives some examples as a guide line, but the final waveforms will eventually depend on the chosen cut-off frequencies of the filters. The low-pass filter will round up the signal because all the high harmonics



(a) Low-pass filter



(b) High-pass filter

Figure 9: First order low-pass and high-pass filters applied on a square wave signal

are filtered out. The high-pass filter enhances the transient steps at the rising and lowering edges of the square wave, so that the high harmonics come out even more distinct compared to the fundamental frequency.

For convenience and for the analysis purposes, The tone section isolated from the Big Muff π schematic is drawn in Figure 10. The component values in the schematic are just one example of a suitable selection. Different versions of the Big Muff have used several different combinations of the resistor and capacitor values.

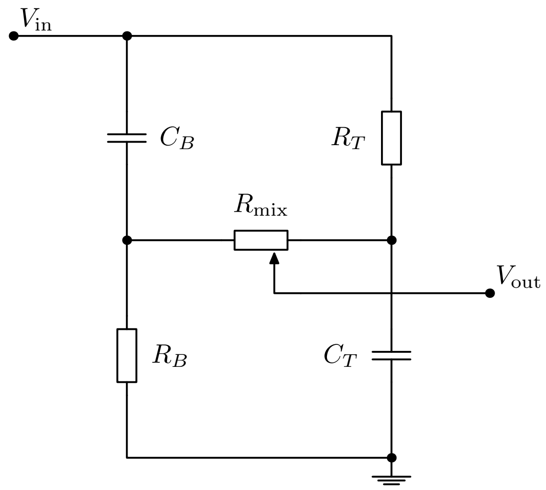


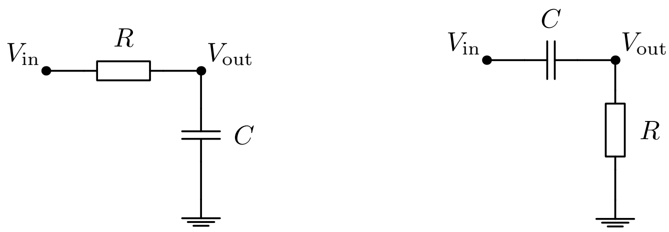
Figure 10: The circuit diagram of the Big Muff π tone control stage

Clearly the circuit has two branches where one branch is a simple high-pass filter and the other one is a simple low-pass filter. The two filters are drawn separately into Figure 11 to clarify how the input and output are related to each other in both filters. The subscript B in the circuit diagram 10 refers to the components on the 'bass-cut' side and the subscript T refers to the components making the 'treble-cut'. These two filters are connected together with a potentiometer, which is used to adjust the filtering somewhere in between the two extremes of the high-pass and low-pass modes.

Although the high-pass and the low-pass sections are not isolated, the filter design can be approached by choosing the cut-off frequencies separately for the high- and low-pass sections. For both configurations, the cut-off frequency (the -3 dB frequency, the corner frequency) is calculated as

$$f_c = \frac{1}{2\pi RC}. \tag{21}$$

Additionally it is good to know that when the mixing resistor is set in the middle, the gap between the high and low cut-off frequencies will determine



(a) Low-pass filter (b) High-pass filter

Figure 11: First order low-pass and high-pass filters

the width of the notch (or pass-band if low cut-off is higher than high cut-off). All this is visualised in Figure 12. Unless someone deliberately wants

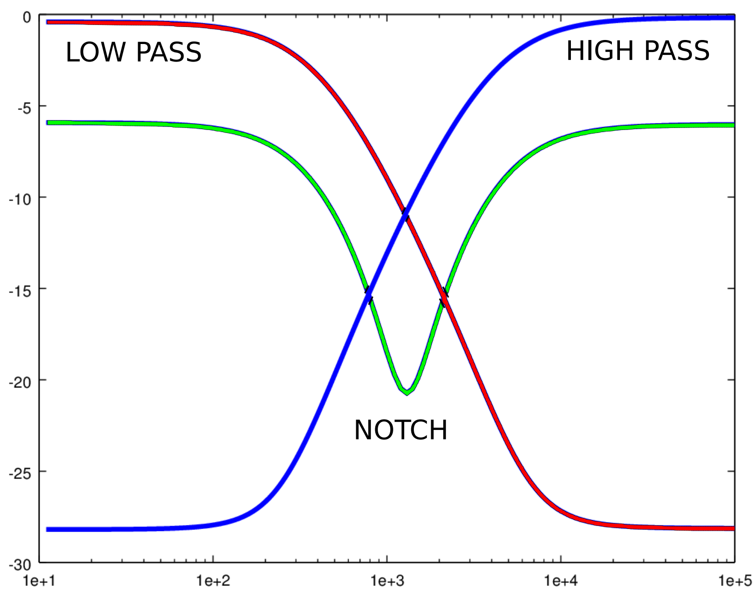


Figure 12: Separated low-pass and high-pass filters and the notch in between

to attenuate a certain range of mid-frequencies out, it is better to design the filter section so that the cut-off frequencies are relatively close together. Then the notch in the middle will be very small, resembling almost like an all-pass filter.

To analyse the complete frequency response of the Big Muff π tone control section, the circuit is redrawn in Figure 13 to prepare for writing the circuit in the matrix form required by the nodal analysis method. The potentiometer has been divided into two separate resistors and the input voltage is split into two current sources to feed both input branches of the circuit.

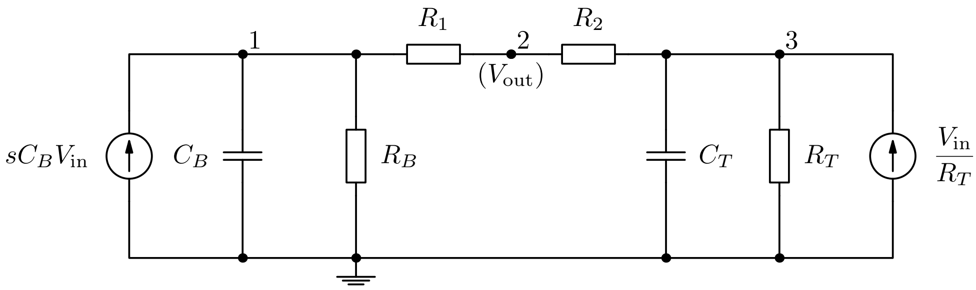


Figure 13: The Big Muff π tone control network redrawn for nodal analysis

The circuit diagram of Figure 13 is represented by a matrix equation shown below, from where all the node voltages and branch currents of the circuit can be solved either symbolically by using Cramer's rule or numerically by using `Octave` or `Matlab`.

$$\begin{bmatrix} \frac{1}{R_B} + \frac{1}{R_1} + sC_B & -\frac{1}{R_1} & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_T} + sC_T \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} sC_BV_{\text{in}} \\ 0 \\ \frac{V_{\text{in}}}{R_T} \end{bmatrix}$$

Since the matrix equation for this circuit is not tremendously huge, the transfer function can be processed as a pen-and-paper calculation using Cramer's rule to solve the expression for the output voltage. After solving the voltage at node 2, the explicit expression for the Big Muff tone control transfer function $H(s) = \frac{V_2}{V_{\text{in}}}$ as a function of the Laplace variable s becomes

$$\frac{1}{R_1 + R_2} \frac{s^2 C_T R_T C_B R_B R_2 + sC_B R_B (R_1 + R_2 + R_T) + (R_1 + R_B)}{s^2 C_T R_T C_B R_B + s\left[C_T R_T + C_B R_B + \frac{R_T R_B}{R_1 + R_2}(C_T + C_B)\right] + \frac{R_T + R_B}{R_1 + R_2} + 1}$$

Hmm, the transfer function looks a lot like the general biquad transfer function

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}, \tag{22}$$

which is called a *biquadratic function* since the numerator and the denominator are both quadratic functions of the Laplacian variable s. This transfer function can be used for modelling second order low-pass, high-pass, band-pass and notch filters. The functional forms of these filters are given in Table 1, where K is a constant gain parameter and ω_0 defines the centre frequency of the band-pass filter or the cut-off frequency of the low/high-pass filters.

The coefficient Q is a quality factor, which indicates how sharply the filter defines the transition between the passband and the stopband. In ideal filters the value of Q is infinitely large, so the bigger the better is the way to go with this parameter.

Table 1: Basic biquad transfer functions

low-pass	*high-pass*	*band-pass*	*notch*
$\dfrac{K}{s^2+\dfrac{\omega_0}{Q}s+\omega_0^2}$	$\dfrac{Ks^2}{s^2+\dfrac{\omega_0}{Q}s+\omega_0^2}$	$\dfrac{Ks}{s^2+\dfrac{\omega_0}{Q}s+\omega_0^2}$	$\dfrac{K(s^2+\omega_r^2)}{s^2+\dfrac{\omega_0}{Q}s+\omega_0^2}$

Generally among filter circuits the width of the passband is measured as a difference of the frequencies at the -3 dB points. In band-pass or notch filters the term $\dfrac{\omega_0}{Q}$ is directly related to the width of the passband. Therefore, the basic properties of biquad filters are quite easily determined by examining the coefficients in the transfer function (22).

Lucky for us, the Big Muff tone control transfer function reveals big secrets on the functionality of the filter. Since R_2 appears only in the term which relates to a high-pass function it is clear that making R_2 larger with respect to R_1 will enhance the high frequencies. Also, since R_1 appears only in the term relating to a low-pass function it will enhance low frequencies. The notch frequency of the filter is found when modifying the equation into the standard form and a good estimate for the notch frequency f_n is given by:

$$f_n = \frac{\sqrt{\dfrac{1+\dfrac{R_T+R_B}{R_1+R_2}}{C_T R_T C_B R_B}}}{2\pi}, \tag{23}$$

which is shared by both high-pass and low-pass forms. Because of the notch, the Big Muff π tone control will attenuate part of the sound in the mid-frequency range. By choosing different values for the mixing potentiometer R_{mix}, one can have a modest control on the notch frequency without altering the actual filter components. However, if the cutoff frequencies of the high- and low-pass filters are close to each other, the notch frequency will be approximately in the middle of the cut-off frequencies (square root of a product with two close numbers is almost the same as taking the average).

To use the transfer function for simulating the frequency response with a sine wave input signal, one can make the substitution $s = j\omega$ as known from the general theory of Laplace transforms. Simulations with the component values shown in the Big Muff π schematic 1 result in a filter function that changes from a low-pass to a high-pass filter as a function of the voltage divider resistors R_1 and R_2. These resistors together form the potentiometer R_{mix} in Figure 10.

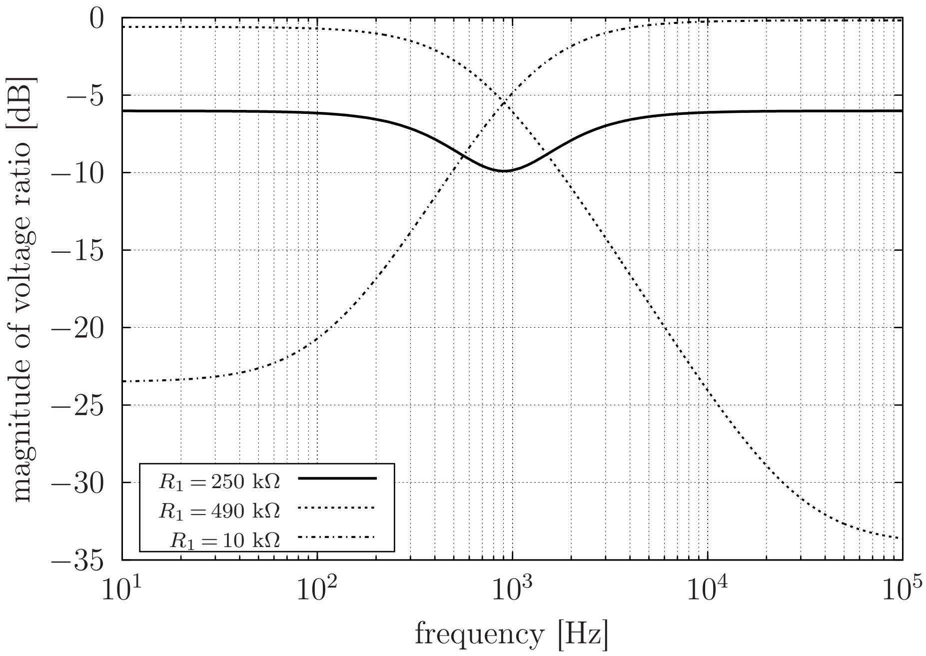


Figure 14: Frequency response curves of the Big Muff π tone control circuit

The frequency response of the Big Muff tone control section indicates that the circuit offers relatively symmetric low-pass and high-pass filtering depending on the resistance of the potentiometer R_{mix}. However, when measured in decibels, the attenuation at the stop-band is not that deep compared to the centre value of -6 dB. When the potentiometer is at the middle position, the notch filter centre frequency is at 900 Hz.

5 Output stage design and analysis

The passive tone control section slightly attenuates the peak-to-peak voltage of the signal. Typically the final circuit stage in audio devices is a buffer with a low output impedance, but also typically after a tone control stage comes a compensating gain stage. I guess here compensation gain has been seen more appropriate than a simple buffer. Also, since guitar effect devices are connected between the guitar and the amplifier, it would be reasonable for the effect device to simulate the output impedance of the guitar. For that purpose a low-impedance buffer is not the preferred choice.

So instead of a low-impedance buffer, the output stage is a medium-impedance amplifier stage; a good compromise. In the Big Muff π the output stage is the most typical four-resistor BJT amplifier, which offers the best possible (but not perfect) stability for quiescent voltages.

Many textbooks offer a standard way to bias the transistor in this configuration, so let's use that method, but simplify it a little bit. For starters select collector resistor R_C to limit collector current and set an approximate values for the output impedance. Then select a value for emitter resistor R_E to set the gain for AC signals. The gain in this configuration is roughly $\frac{R_C}{R_E}$. Next we follow the 'standard' design rule for this configuration to find the values for the base-bias resistors; the rule is that the parallel resistance $R_{B1}||R_{B2}$ should be approximately equal to $0.1(\beta+1)R_E$. Basically this rule comes from similar reasoning as the earlier presented method for biasing the collector-base feedback transistor configuration. The 1/10 rule, that is. Following the given design rule, one can derive an analytic equation for R_{B1}, which is

$$R_{B1} = \frac{0.1(\beta+1)R_E V_{CC}}{V_{BE} + 1.1(\beta+1)R_E \frac{I_C}{\beta}},$$

and once R_{B1} is known,

$$R_{B2} = \frac{0.1(\beta+1)R_E}{1 - 0.1(\beta+1)\frac{R_E}{R_{B1}}}.$$

In the above equations the collector bias current is obtained from the equation

$$I_C = \frac{V_{CC} - V_C}{R_C},$$

where V_C is the chosen collector quiescent voltage. As usual, the obtained base resistor values need to be rounded to the nearest standard resistor values.

Then after the design is ready, one should make sure that the obtained component values are valid. To see if the obtained values give any meaningful results, just plug the values into the equation

$$V_C = V_{CC} - \beta_F R_C \frac{V_{CC}\frac{R_{B2}}{R_{B1}+R_{B2}} - V_{BE}}{\frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}} + (\beta_F+1)R_E}. \tag{24}$$

From this equation one should get very close to the collector quiescent voltage, which was chosen during the design phase.

6 Summarising the results

AS a general summary we will just step through the Big Muff π circuit of Figure 1 component by component:

R_{G1} sets AC gain with R_{B11}, voltage gain is approximately $\frac{R_{B11}}{R_{G1}}$.

C_{B1} DC blocker, forms an (unintentional) high-pass filter with R_{B12}

R_{B11} sets the bias voltage divider with R_{B12}. Controls AC gain with R_{G1}.

R_{B12} sets the bias voltage divider with R_{B11}. Forms high-pass filter with C_{B1}.

C_{F1} parallel AC impedance with R_{B11}, creates higher impedance (and higher AC gain) on low frequencies

R_{C1} limits collector current on DC biasing and affects stage 1 output impedance

R_{E1} adds second feedback loop to stabilise DC bias. Profit is small.

C_{O1} DC blocker, again minor high-pass filtering with R_{sus}

R_{sus} voltage divider to reduce signal level obtained from gain stage 1

C_{B2} another DC blocker, because without it R_{G2} would affect biasing

R_{G2} sets AC gain with R_{B21}, voltage gain is approximately $\frac{R_{B21}}{R_{G2}}$

R_{B21} sets the bias voltage divider with R_{B22}. Controls AC gain with R_{G2}.

R_{B22} sets the bias voltage divider with R_{B21}

C_{F2} parallel AC impedance with R_{B21}, creates higher impedance (and higher AC gain) on low frequencies.

R_{C2} limits collector current on DC biasing and affects stage 2 output impedance

R_{E2} adds second feedback loop to increase DC bias. Profit is small.

D_{21} clips collector AC voltage to $V_C + V_g$

D_{22} clips collector AC voltage to $V_C - V_g$

C_{R2} DC blocker, works with the diodes to set DC level to $V_C \pm V_g$

C_{O2} DC blocker

R_{G3} sets AC gain with R_{B31}, voltage gain is approximately $\frac{R_{B31}}{R_{G3}}$

R_{B31} sets the bias voltage divider with R_{B32}. Controls AC gain with R_{G3}.

R_{B32} sets the bias voltage divider with R_{B31}

C_{F3} parallel AC impedance with R_{B31}, creates higher impedance (and higher AC gain) on low frequencies.

R_{C3} limits collector current on DC biasing and affects stage 3 output impedance

R_{E3} adds second feedback loop to stabilise DC bias. Profit is small.

D_{31} clips collector AC voltage to $V_C + V_g$

D_{32} clips collector AC voltage to $V_C - V_g$

C_{R3} DC blocker, works with the diodes to set DC level to $V_C \pm V_g$

R_T with C_T forms a low-pass filter to perform treble-cut

C_T with R_T forms a low-pass filter to perform treble-cut

R_B with C_B forms a high-pass filter to perform bass-cut

C_B with R_B forms a high-pass filter to perform bass-cut

R_{mix} voltage divider that mixes between treble-cut and bass-cut

C_{B4} DC blocker, forms an (unintentional) high-pass filter with $R_{B41}||R_{B42}$

R_{B41} sets the bias voltage divider with R_{B42}. In AC parallel to R_{B42}. Forms high-pass filter with C_{B4}.

R_{B42} sets the bias voltage divider with R_{B41}. In AC parallel to R_{B41}. Forms high-pass filter with C_{B4}.

R_{C4} limits collector current on DC biasing and affects stage 4 output impedance. Sets gain with R_{E4}

R_{E4} adds a feedback loop to stabilise DC bias. Sets gain with R_{C4}.

C_{O4} DC blocker

R_{O4} voltage divider, output impedance.